Supply Chain Resilience:
Should Policy Promote Diversification or Reshoring?*

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Abstract

Little is known about optimal policy in the face of potential supply chain disruptions. Should governments promote resilience by subsidizing backup sources of input supply or encourage firms to source from safer, domestic suppliers? We address these questions in a model of production with a single critical input and exogenous risks of relationship-specific and country-wide supply disturbances. In the CES case, a subsidy for diversification achieves the constrained social optimum. When the demand elasticity rises with price, private investments in resilience may be socially excessive and the social planner may wish to favor sourcing at home or abroad.

Keywords: global supply chains, global value chains, input sourcing, resilience,

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1 Introduction

> The United States needs resilient, diverse, and secure supply chains to ensure our economic prosperity and national security. Pandemics and other biological threats, cyber-attacks, climate shocks and extreme weather events, terrorist attacks, geopolitical and economic competition, and other conditions can reduce critical manufacturing capacity and the availability and integrity of critical goods, products, and services. Resilient American supply chains will revitalize and rebuild domestic manufacturing capacity, maintain America's competitive edge in research and development, and create well-paying jobs.
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> Joseph R. Biden, Jr., Executive Order on America's Supply Chains, February 24, 2021

Supply chain disruptions have become the new normal. The Great East Japan Earthquake of 2011, and the massive tsunami that it triggered, brought such occurrences to the attention of economists. Since then, hardly a month passes without news of a fresh disturbance. The pace of disruptions has quickened with the advent of the COVID-19 pandemic, and now we hear regularly of supply chain breakdowns in industries as disparate as automobiles, dishwashers, plastics, copper wire, lumber, pork, and toilet paper.

Disruptions have a myriad of causes. They result from natural disasters, geopolitical disputes, transportation failures, cyber-attacks, fires, power outages, labor shortages, human error and, of course, pandemics. McKinsey Global Institute (2020), which recently conducted a series of interviews with supply chain experts, reports that disruptions lasting one to two weeks happen to a given company on average every second year, while those lasting one to two months occur every 3.7 years. They find that an industry's exposure to shocks reflects its geographic footprint and its production technology, with greatest exposure in medical devices, wooden products, and fabricated metal products and least exposure in communications equipment, apparel, and petroleum products (see Exhibit E2 on p.6). The disruptions impose significant costs, presenting firms with expected losses of between 24% of a year's earnings before interest, taxes, depreciation and amortization (EBITDA) in pharmaceuticals to 67% in aerospace, over a ten-year period. Across the thirteen industries that McKinsey examined, the expected losses per decade amounted to about 42% of annual EBITDA (see Exhibit E5 on p.12).

Carvalho et al. (2021) provide a more rigorous quantitative assessment of one major disruption, namely the aforementioned Japanese earthquake of 2011. They focus on the role of input-output linkages as a mechanism for propagation and amplification of shocks, exploiting the exogenous nature of the event, its enormous impact on a subset of firms, and its localized incidence. Beyond the direct effects, they find that firms whose suppliers were hit hard by the natural disaster suffered substantial sale losses compared to others, as did firms whose downstream customers were hit. Using a calibrated general-equilibrium model of production networks, they conclude that the disaster imposed a 0.47 percentage point reduction in Japan's aggregate real GDP growth.

Many commentators associate the increasing frequency and severity of supply chain disruptions with the recent history of rapid globalization.[1] This perceived connection, in turn, has sparked soul searching amongst policy makers and a call to action in the broader public. If costly shocks reflect concentration of input supplies in a few regions or countries, wouldn't it be sensible for governments to encourage firms to diversify their sourcing? And if distance from suppliers intensifies the risk of disruption, wouldn't it be better to bring some parts of the supply chains nearer to home? The preamble to President Biden's February 24th 2021 Executive Order suggests that "*diverse* and secure supply chains" are a prerequisite for economic prosperity and that "resilient American supply chains" will rest on "rebuil[t] *domestic* manufacturing capacity [emphasis added]."

Little is known about the efficacy of policies aimed at supply chain management in an environment with recurring disturbances. Disruptions generate input shortages that can give rise to price spikes or even outright unavailability of downstream products. Consumers suffer from their hampered ability to purchase the products they covet. To the extent that households forfeit consumer surplus in the face of supply chain disruptions, governments may have reason to enact policies that curtail their occurrence.

But the policy calculus is not so simple. Production impediments impact not only consumers' surplus, but also firms' bottom lines. The question for governments is not whether shortages adversely affect households, but whether firms' private incentives to avoid such shortages fall short of (or exceed) what is socially desirable. Firms may have inadequate incentives to invest in supply chain resilience, because they do not capture all of the surplus from offering their products to the market. But they may also have an excessive taste for resilience arising from their desire to be in a position to capitalize on extraordinary profit opportunities when their rivals are hit with supply problems of their own.

In this paper, we propose a bare-bones framework that can aid with evaluating policies that influence supply chain organization. Our framework puts supply shortages front and center. We abstract from all complexity in the production process by assuming that each firm manufactures a unique variety of some good using a single, critical input. If the supply chain operates smoothly, the firm produces one unit of its variety from one unit of the customized input. But exogenous shocks may disrupt supplier-buyer relationships. We allow for two types of shocks, those that idiosyncratically sever a single chain and those that impede all supply from a particular source region. Each firm may establish a relationship with a potential supplier in a low-cost but riskier foreign country, in a higher-cost but safer home location, or in both. To form a relationship with some supplier, the firm bears a fixed cost. A firm can invest in resilience by avoiding the riskier foreign supply or, even more so, by diversifying its supply base by establishing relationships in both countries. In equilibrium, there are four possible states of the aggregate economy: supply chains may be operative only with home suppliers, only with foreign suppliers, with neither, or with both. The fixed mass of final producers chooses among four strategies: invest in a single supply relationship domestically, invest in a single relationship abroad, diversify, or exit. Their collective choices determine equilibrium prices, equilibrium variety, profits, surplus and (with fiscal policies

[1] See, for example, Shih (2020a, 2020b) and Iakovou and White III (2020).

in place) government revenues in each state of the world. Using these state-contingent aggregate outcomes, we can calculate expected welfare under different policy regimes.

Since firms have three options in forming their supply chains (besides exit), we endow the social planner with three policy instruments to influence chain organization that we link directly to the available strategies. The planner can encourage or discourage local procurement with a payment or levy to any firm that develops a supply relationship only in the home country. She can encourage or discourage offshoring with a subsidy or tax to any firm that forms a supply relationship only in the foreign country. And she can encourage or discourage supply chain resilience with a payment or tax to any firm that diversifies its sourcing opportunities by investing in relationships in both countries. These options afford the planner a degree of freedom inasmuch as she only needs to influence two margins of a firm's behavior: whether it should invest in one relationship or two, and conditional on the former, whether that supplier should be at home or abroad. To maximize expected welfare, the planner can use any two of the three available instruments, or various linear combinations of the three.[2]

Our policy environment is one with monopoly pricing of differentiated products alongside competitive pricing of a numeraire good. It is well known that, in such settings, relative prices in the two sectors do not mirror relative marginal costs and so the allocation of spending across broad aggregates is not efficient in a *laissez-faire* equilibrium (see, for example, Dhingra and Morrow, 2019, and Campolmi et al., 2021). In principle, the distortion generated by monopoly pricing can be addressed with a set of optimal consumption subsidies, but such subsidies are often viewed as unrealistic. In our setting with multiple states of the world that arise according to the realization of the supply shocks and with endogenous markups, the optimal consumption subsidies would need to be state-and-product specific. Although we consider the implementation of such consumption subsidies to be impractical, we nonetheless begin our analysis with a planner's problem that allows for such subsidies in order to focus most sharply on the divergence of private and social incentives for supply chain organization. Once we understand this divergence, we can turn to the second-best planner's problem that arises when consumption subsidies are infeasible.

Inasmuch as the social cost of supply chain disruptions stems from loss of consumer surplus, the form of consumer preferences plays a crucial role in our policy analysis. It has become commonplace to use CES preferences in trade models with endogenous availability of differentiated products, but the very special properties of these preferences have been recognized since the seminal work by Dixit and Stiglitz (1977). The market undersupplies variety, because firms do not capture all of the surplus from their investments. But it oversupplies variety, because some of a firm's profits come at the expense of rival firms. In many contexts with CES preferences, these opposing forces happen exactly to offset one another. These considerations apply as much to investments in "resilience" as they do to investments in entry, so it is important for understanding the dictates of efficient organization of supply chains to allow for more flexible forms of demand. To this end, we

[2]We could also specify the policies differently, without affecting the outcomes. For example, the planner could stipulate a payment or levy for every relationship a firm forms at home and a payment or levy to any firm that diversifies across borders.

follow Matsuyama and Ushchev (2017, 2020a) in adopting a broader class of preferences that are *Homothetic with a Single Aggregator* (HSA). With HSA preferences, the demand for any variety per unit of income depends on the price of that variety relative to a (common) aggregator of all prices. The CES utility function is a member of this class wherein the ideal price index plays a dual role of inversely measuring welfare and capturing the competitive pressure from rival varieties. More generally, HSA preferences allow the demand elasticity for a variety to vary along the demand curve and the aggregator that enters demand to differ from the one that measures welfare.

We find that, in general, profit incentives for the organization of supply chains yield socially inefficient outcomes. With CES preferences and optimal subsidies to address consumption distortions arising from markup pricing, the planner need not encourage nor discourage diversification, nor must she alter incentives for sourcing at home versus abroad. But with more general forms of HSA preferences that obey Marshall's Second Law of Demand, she requires not only consumption subsidies that vary across states of the world and according to the source of inputs, but also a policy to discourage diversification and another to alter the incentives to form chains at offshore or onshore. To achieve the second best when consumption subsidies are infeasible, CES preferences dictate a subsidy for diversification as the only necessary supply chain policy. However, with more general HSA preferences, the second-best policy might be to encourage or discourage diversification, according to how the excess private incentive for resilience that reflects a "business-stealing motive" compares to the shortfall in consumption of differentiated products that occurs due to monopoly pricing.

Needless to say, there are many ways that our analysis could be enriched beyond allowing for a broader class of demands than CES. For example, we could introduce a richer technology with potential substitution between manufactured inputs and primary factors of production. We could entertain more complex supply chains, with multiple inputs and with a sequencing of them such that some inputs enter the production process upstream from others. We could allow for dynamics, which would render inventories an additional tool for firms to invest in resilience and give governments additional policy instruments such as stockpiling supplies or allowing accelerated depreciation of inventory costs. We could introduce political-economy considerations that might drive a wedge between the parameters that capture the risk aversion of managers versus that of policy makers. We see all of these potential extensions as worthwhile and germane to the ultimate policy assessment. Our simpler setting suggests a way to pose the question and our analysis provides a "proof of concept."

Our paper fits into an earlier literature on trade disruptions in a neoclassical setting. Much of this previous work addressed optimal policy responses to potential trade embargoes. Mayer (1977) showed that production subsidies are an optimal response to threats of trade interruption in the presence of costly adjustment. Bhagwati and Srinivasan (1976) made the likelihood of a disruption a function of the volume of trade and elucidated an efficiency role for tariffs to give agents an incentive to internalize the externality arising from their effect on the probability of a trade restriction. Arad and Hillman (1979) extended these earlier papers to allow for learning-by-doing in the production of a good that might later be subject to an embargo. Bergström et al. (1985) developed an infinite-horizon model to study the potential role of inventories to mitigate the threat of embargo. Perhaps the most sophisticated of these early studies was that by Cheng (1989), who considered recurrent embargo threats as a two-state stationary Markov process that plays out with constraints on the speed of intersectoral reallocation.

The main difference between our work and this earlier literature stems from our treatment of the endogenous availability of differentiated products. With perfect competition and homogeneous goods, aggregate quantities matter for welfare but the availability of a particular firm's offering does not. If a disruption causes some import good to be unavailable, there is no harm to consumers beyond the higher price of the domestic (perfect) substitutes. The number of viable producers plays no role in neoclassical welfare analysis and is not even well defined. Of course, higher sticker prices play a role also in a world with differentiated products, but there is also a direct harm to consumers from a particular variety not being available for purchase. For this reason, we believe that endogenous determination of the set of available products should feature prominently when evaluating policy toward supply-chain security.

Our paper also relates to a literature on distortions in the entry process, which began with the seminal paper by Dixit and Stiglitz (1977) and includes more recent contributions by Bilbiie et al. (2012), Matsuyama and Ushchev (2020a), Baqaee and Farhi (2020), among others. This literature focuses on entry of new firms into an industry in settings with imperfect competition. Bilbiie et al. (2012) study pricing and entry over the business cycle, focusing on intertemporal variation in markups and their relation to intertemporal marginal rates of substitutions. Baqaee and Farhi (2020) decompose the welfare losses from inefficient pricing and entry by calculating second-order approximations around an efficient equilibrium, while mostly treating markups as given. Matsuyama and Ushchev (2020a) analyze the inefficiencies that arise in a one-sector model with endogenous entry and endogenous markups. They introduce HSA preferences to allow for non-constant markups and Marshall's Second Law of Demand, an original approach that proves very useful in our context as well. Our paper differs from this literature inasmuch as we do not consider new entry into an industry, but rather the organization of supply chains that determines product availability in different states of the world. Unlike Baqaee and Farhi (2020), we derive exact wedges that describe the gaps between social valuations and private valuation and use these wedges to characterize first-best and second-best policies. Our model is designed to address inefficiencies in *global* value chains in the face of supply disruptions and so we emphasize the asymmetries in cost and risk that often characterize international trade in intermediate goods.[3]

The remainder of this paper is organized as follows. In Section 2, we develop our model of risky supply chains in which differentiated final goods are produced with a single, critical input that may be subject to relationship-specific and country-wide supply shocks. Once the disturbances are realized, firms that have access to their essential inputs engage in monopolistic competition

[3] A tangentially related paper is Elliot et al. (2020), who study networks with idiosyncratic probabilities of breakup that depend on investment choices. They do not entertain macro shocks as here, except in some comparative statics. Rather, they are interested in the propogation of idiosyncratic shocks up and down the supply chain.

alongside the competitive pricing of a numeraire good. The *laissez-faire* equilibrium described in Section 3 determines state-contingent price indexes and welfare. In Section 4, we pose and solve the social planner's problem that arises when she has access to state-and-product specific consumption subsidies to offset the inefficiencies caused by monopoly pricing. We characterize the "wedges" between social and private incentives for forming supply relationships in each country relative to an alternative of diversified sourcing. The wedges identify the set of supply-chain policies that can be used to achieve the first-best allocation.

Armed with this understanding of the sources of inefficiency in supply-chain formation, we turn in Section 5 to the more realistic policy problem that arises when consumption subsidies are infeasible. Then optimal supply chain policy must be used to address not only the "consumer-surplus externality" that arises because a firm does not capture all the surplus from the availability of its product and the "business-stealing externality" that arises because a firm does not consider the impact of its product's availability on the profits of others, but also the distortion associated with markup pricing. We fully characterize the supply chain policies that achieve a constrained optimum for arbitrary cost and risk parameters when preferences take the CES form. For more general, HSA preferences, our theoretical results apply only when cost and risk parameters are not very different in the two countries. So, to better understand the forces at work when foreign suppliers are significantly cheaper but also riskier than their domestic counterparts, we resort in Section 6 to numerical methods. Section 7 concludes. An appendix contains further details, proofs, and additional numerical simulations.

2 A Simple Model of Risky Supply Chains

2.1 Supply Relationships

The home economy can produce a homogeneous, numeraire good and potentially a unit measure of nontraded differentiated final products. Revenues from sales of the numeraire good amount to $\bar{Y}$, all of which is paid to workers as labor income. Production of differentiated products requires no labor. Rather, firm ω in this industry converts a single, customized critical input into the final good ω using the linear production technology,

$$x(\omega) = m(\omega),$$

where $x(\omega)$ is output of good ω and $m(\omega)$ is the quantity of the customized input. If the firm has established a supplier relationship in country i and if that supply chain is operative, then the firm can procure the customized inputs at a cost of q_i per unit, $i = H, F$, where the subscripts denote "home" and "foreign," respectively, and we assume that $q_F < q_H$. This is, of course, the simplest imaginable production function; in future work, we plan to allow for additional factors of production and more complex supply chains. But the linear production technology provides a good starting point.

The constant cost of procurement fits best with a market structure in which the downstream firm is vertically integrated with its upstream subsidiaries. Alternatively, we can imagine a situation in which the downstream firm has all the bargaining power in its relationship with arms-length suppliers. If the price of the input results instead from a bilateral negotiation, this might provide an additional reason for the firm to diversify its sourcing in order to strengthen its hand in bargaining. Our setup allows us to focus most sharply on supply disruptions, leaving other potential motives for diversification for future research.

To form any supply relationship, a firm must bear a sunk investment cost, k, in units of the numeraire good. This cost captures the up-front outlays associated with searching for a partner, negotiating a contract, and designing a suitable input. Once a supply relationship has been established, it is subject to two possible "disruption shocks." With probability $1-\rho$, any particular supply chain breaks down for exogenous and idiosyncratic reasons, which might be a failure of the prototype input, a strike in the supplier factory, a localized weather event in the location where the input would be produced, or anything else that happens independently of all other supply relationships. In any of these circumstances, the downstream firm loses the ability to purchase its input from the particular supplier for the length of the period captured by our model.[4] In the complementary event, with probability ρ, no idiosyncratic supply disruption occurs and the firm can buy as much as it wants from the particular supplier provided that the latter is located in a country that is "open for business."[5] However, with probability $1-\gamma_i$ a country-wide shock disrupts all chains with suppliers in country i. These shocks, which we assume to be independent across countries (to simplify the expressions, but with no substantive importance), represent events such as epidemics, political conflicts between national governments, or failures of the national transportation system. The relative safety of the home country is captured by the assumption that $\gamma_H > \gamma_F$.

The realizations of the two country-wide shocks generate four possible (macro) states of the world that we denote by $J \in \{H, F, B, N\}$. In state H, all foreign sources of supply are disrupted and only a firm that has a supplier located in the home country might operate, conditional on its relationship surviving any idiosyncratic shock. In state F, it is the home market that is disrupted, and only firms with foreign suppliers might operate. In state B, no broad-based disruptions occur and firms that escape idiosyncratic shocks to all of their suppliers will be active in the market. Finally, in state N, both countries suffer adverse shocks and no production of differentiated products takes place.

After the realization of the supply shocks, firms produce their varieties if they can. A firm that has invested in a single supply relationship in country i operates with probability $\gamma_i\rho$ at unit

[4]We treat all disruptions as catastrophic; when they occur, they eliminate all input supply from the affected source. Alternatively, we could allow for less severe shocks that limit supply to some positive quantity or that raise the cost of purchases above q_i.

[5]Our analysis could be conducted without the idiosyncratic shocks, i.e., with $\rho = 1$. However, $\rho < 1$ provides an additional incentive for diversification, and it implies that not all firms will be able to operate even when neither country suffers a broad-based disruption. Moreover, the number of active firms in the state of the world when neither source country is fully disrupted will depend on the sourcing strategies adopted by the firms, which then becomes a consideration in the choice of policy.

cost q_i, whereas its product becomes unavailable in the face of any disruption, which happens with probability $1-\gamma_i\rho$. A firm that instead pursues a strategy of diversification can produce if at least one of its supply chains remains viable. It prefers to produce at the lower unit cost q_F, which it can do with probability $\gamma_F\rho$. Should its offshore relationship be disturbed, it can turn to its home supplier with probability $\gamma_H\rho$. Therefore, the unconditional probability that it produces at cost q_H is $\gamma_H\rho\,(1-\gamma_F\rho)$. Finally, with probability $(1-\gamma_F\rho)\,(1-\gamma_H\rho)$ it will find both of its potential supply chains disrupted and it will be unable to serve the market.[6]

2.2 Preferences and Demand

There is a unit mass of identical consumers in the home country. The representative consumer holds quasi-linear preferences over consumption of the homogeneous good, Y, and consumption of differentiated products, indexed by X, so that total utility is given by

$$V\left(X,Y\right)=Y+U\left(X\right), \tag{1}$$

where $U\left(\cdot\right)$ has a constant elasticity $\varepsilon>1$;[7] i.e.,

$$U\left(X\right)=\frac{\varepsilon}{\varepsilon-1}\left(X^{\frac{\varepsilon-1}{\varepsilon}}-1\right)\qquad\text{for }\varepsilon>1.$$

The representative consumer maximizes her utility in any state of the world subject to a standard budget constraint, $Y+\int_{\omega\in\Omega}p\left(\omega\right)x\left(\omega\right)d\omega=I$, where $p\left(\omega\right)$ is the price and $x\left(\omega\right)$ the quantity purchased of variety ω, Ω is the set of varieties available in the relevant state of the world, and I is her income. The constant elasticity of $U\left(X\right)$ gives rise to a constant-elasticity demand for differentiated products,

$$X=P^{-\varepsilon}, \tag{2}$$

where P is the real price index dual to U. The consumer spends $PX=P^{1-\varepsilon}$ on differentiated products and devotes residual spending of $I-P^{1-\varepsilon}$ to the homogeneous good.

Following Matsuyama and Ushchev (2017, 2020a), we assume that preferences for the bundle of differentiated products belong to a class they aptly term *Homothetic with a Single Aggregator* (or HSA). Homotheticity implies that the consumption index X is a linearly homogenous function of consumption of the individual varieties $\{x\left(\omega\right)\}_{\omega\in\Omega}$. A single aggregator, A, which is a linearly homogenous function of the set of prices $\{p\left(\omega\right)\}_{\omega\in\Omega}$, guides the substitution between a particular variety ω and all other varieties. More formally, HSA preferences require the existence of a price aggregator A and market-share function $s\left[p\left(\omega\right)/A\right]$ that is non-negative for all relative prices such

[6]In principle, a firm that diversifies may choose to invest in multiple supply relationships in the same country. To avoid a taxonomy, we do not consider this possibility here; it will not be an attractive option for ρ close to one.

[7]A unitary elasticity can be treated as a limiting case as $\varepsilon\to1$. We cannot allow $\varepsilon=1$, because then $V\to-\infty$ in the state of the world when all firms face supply disruptions in both countries. We could introduce a backup technology such that firms can produce their own critical inputs at some high cost $\bar{q}>q_H$ and then we could entertain $\varepsilon=1$ and even $\varepsilon<1$. But introducing the additional parameter $\bar{q}$ complicates the expressions without providing additional insights. Accordingly, we choose to restrict the range of the demand elasticity.

that

$$\frac{d\log P}{d\log p(\omega)} = s\left[\frac{p(\omega)}{A}\right] \tag{3}$$

and

$$\int_{\omega\in\Omega} s\left[\frac{p(\omega)}{A}\right] d\omega = 1\ . \tag{4}$$

Equation (3) expresses the demand for any variety ω in implicit form; the substantive assumption is that this demand depends only on the price of variety ω relative to a *common* aggregator, namely $z(\omega) := p(\omega)/A$. Equation (4) stipulates that the market shares sum to one.

We place some mild restrictions on the market-share function, $s(z)$. First, we impose

Assumption 1 *The market-share function $s(z)$ is strictly decreasing when positive, with $\lim_{z\to 0} s(z) = \infty$ and $\lim_{z\to\bar{z}} s(z) = 0$, for $\bar{z} \equiv \inf\{z>0|\, s(z)=0\}$.*

The assumption that $s(z)$ is strictly decreasing ensures that all varieties in X are gross substitutes. It admits both the case when $\bar{z} < \infty$, so that demand "chokes off" at some finite relative price, and the case $\bar{z} = \infty$, when positive quantities are demanded at any finite price. The assumption that $s(z) \to \infty$ as the relative price goes to zero is a simple way to ensure that the aggregator A is well defined for any measure of firms.[8]

Equation (3) implies that the elasticity of substitution between any two goods with equal prices is a function of the common relative price, and is given by

$$\sigma(z) = 1 - \frac{zs'(z)}{s(z)} > 1.$$

We also adopt

Assumption 2 *(i) Either $\sigma(z) = \sigma$ or $\sigma'(z) > 0$ for all $z \in (0,\bar{z})$ and (ii) $\sigma(z) > \varepsilon$ in the neighborhood of the equilibrium and the social optimum.*

The first part of Assumption 2 allows for the case of *Symmetric CES* preferences, where $s(z) = \alpha z^{1-\sigma}$, $\alpha > 0$, and the aggregator A is proportional to the price index P. For all other HSA preferences, we impose *Marshall's Second Law of Demand (MSLD)*, namely that the demand for a good becomes more elastic as its price rises.[9] For example, *Symmetric Translog* preferences, developed by Feenstra (2003), drawing on Diewert (1974), satisfy MSLD. These preferences can be

[8]The assumption that $\lim_{z\to 0} s(z) = \infty$ is satisfied for all parameters of the CES and symmetric translog functions that we use as examples of HSA below, but that is not so for all parametric families of HSA preferences, including what Matsuyama and Ushchev (2020b, 2022) term Constant Pass Through (or CoPaTh) preferences. They discuss alternative assumptions that would ensure a well-defined aggregator, such as, for example, limiting the size of the market relative to the fixed cost of entry.

[9]Zhelobodko et al. (2012) describe this assumption as increasing "relative love of variety" while Mrázová and Neary (2017) refer to it as the case of "sub-convex" demand. One advantage of the HSA class of utility functions relative to the ones that these and other authors have used to allow for a non-constant elasticity of substitution is that it does not impose additive separability of preferences. Additive separability implies a tight relationship between the price and income elasticities of demand that rules out MSLD for homothetic preferences. See Matsuyama (2019) for further discussion.

represented by a market-share function $s(z) = -\theta \log z$, $z \in (0,1)$, $\theta > 0$. Then $\sigma(z) = 1 - 1/\log z$. For this specification,

$$\log A = \frac{1}{\theta n} + \frac{1}{n} \int_{\omega \in \Omega} \log p(\omega)\, d\omega\ ,$$

where $n := \int_{\omega \in \Omega} d\omega$ is the measure of products available on the market. Here, the aggregator A that enters demands differs from the price index P that enters the indirect utility function. More generally, Matsuyama and Ushchev (2020a) prove that the alternative price aggregates are related by

$$\log P = C_P + \log A - \int_{\omega \in \Omega} \int_{p(\omega)/A}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta d\omega, \tag{5}$$

where C_P is a constant.

The second part of Assumption 2 ensures that the demand for any variety ω increases when the aggregate price of competitor brands rises. For some market-share functions, this assumption might be satisfied at all values of $z \in (0, \bar{z})$. For others, we would need to verify that it is satisfied ex post, i.e., after solving for the equilibrium.

2.3 Profit Maximization

The identical households collectively own the unit measure of downstream (potential) producers. Since the quasi-linear utility represented by (1) implies that these households are risk neutral with respect to income shocks, the firms make their ex-ante investment to maximize expected profits. Once the supply shocks are realized, the surviving producers purchase inputs and set prices to maximize profits taking into account the competition they face in the realized state of the world.

The firm producing variety ω maximizes profits in any state by procuring its inputs at minimum cost and by marking up price over the relevant marginal cost, q_J, $J \in \{H, F\}$. Specifically, a firm that pays q_J for its inputs solves

$$p(\omega) = \arg\max_{\mathsf{p}} P^{1-\varepsilon} s\left(\frac{\mathsf{p}}{A}\right) \mathsf{p}^{-1} [\mathsf{p} - q_J]\,,$$

taking the state-contingent price index P and the state-contingent aggregator A as given. Profit maximization requires

$$p(\omega) = \frac{\sigma[z(\omega)]}{\sigma[z(\omega)] - 1} q_J \tag{6}$$

and yields operating profits

$$\pi(\omega) = \frac{s[z(\omega)]}{\sigma[z(\omega)]} P^{1-\varepsilon}. \tag{7}$$

Notice that the price of any variety might vary across states of the world and the source of its inputs. The markup reflects the elasticity of demand, as usual, but the latter may not be constant or independent of the state.

2.4 Supply Chain Management

We allow firms to choose among three modes of organization (plus exit). Strategy h entails investment in a single supply relationship in the home country in the hope of “onshoring.” Strategy f entails investment in a single relationship in the foreign country in the hope of “offshoring.” Strategy b (for “both”) involves diversification, i.e., investment in supply relationships in both places with the intention of sourcing from the low-cost foreign supplier if that is possible, and from the higher-cost domestic supplier if that is possible and the low-cost foreign option is not available. As we noted above, firms organize their supply chains to maximize expected profits.

Firms calculate expected profits with rational expectations about prices, sales, and costs in each state of the world, in view of the fraction of their competitors that pursue each strategy in equilibrium. Let μ_j be the fraction of firms that opt for strategy j, $j \in \{h, f, b\}$, with $\sum_j \mu_j \leq 1$. In state H, when all foreign sources of supply are disrupted, only firms that have chosen strategy h or strategy b might be operative, and among those, only the ones that avoid an idiosyncratic shock. Each such firm faces competition from $\rho\,(\mu_h + \mu_b)$ others, all of which have a unit cost of q_H. This state occurs with probability $\gamma_H\,(1-\gamma_F)$. Analogously, in state F, it is the firms that pursued strategy f or strategy b that might produce. Again, only a fraction ρ can do so, because the others suffer relationship-specific supply disturbances. It follows that in state F, an active firm competes with $\rho\,(\mu_f + \mu_b)$ others, each of which has a unit cost of q_F. State F arises with probability $\gamma_F\,(1-\gamma_H)$.

A firm’s expected profit calculation for state B in which supply chains in both countries are active is slightly more complicated. In this state, firms that adopt either strategy f or strategy b anticipate a cost of q_F with probability ρ. Those that diversify by choosing strategy b anticipate that they will rely on their backup supplier, at the higher cost q_H, with probability $\rho\,(1-\rho)$. Meanwhile, firms that pursue strategy h also produce at q_H, but with probability ρ. It follows that all firms anticipate competition in state B from $(\mu_f + \mu_b)\,\rho$ others producing at cost q_F and from $\mu_h\rho + \mu_b\rho\,(1-\rho)$ others producing at cost q_H. State B occurs with probability $\gamma_H\gamma_F$.[10]

In Section 2 of the appendix, we tally in (32)-(34) the expected profits associated with each strategy as a function of the fractions of others in the industry that make the various choices.[11] We denote these expected profit opportunities by Π_h, Π_f, and Π_b for strategies h, f and b, respectively. In equilibrium, if there is one dominant strategy j, all active firms will make that choice, and so $\mu_j \leq 1$, while $\mu_\ell = 0$ for $\ell \neq j$. If two strategies yield equally high expected profits and higher than the third, then these two can have any positive fractions in equilibrium, while the third will find no takers. The fractions will be such as to generate indifference. Finally, if there exist $\mu_h > 0$, $\mu_f > 0$ and $\mu_b > 0$ such that $\Pi_h = \Pi_f = \Pi_b$, then the equilibrium will exhibit a positive number of firms pursuing each of the available strategies.

[10]Of course, operating profits are zero in state N, when no firm can produce.

[11]Section numbers in the appendix correspond to the sections in the main text. Henceforth, when we refer to the appendix for formal arguments, we will only note the section number when it is an exception to this general rule.

2.5 Welfare

We adopt expected indirect utility as our welfare metric, weighting utility in each aggregate state by the likelihood of that state. Indirect utility comprises labor income, profits, tax revenues (if any) and consumer surplus.

Expected welfare reflects the fractions of firms that choose each organizational mode, outcomes that can be influenced by government policy. When μ_h firms adopt strategy h, μ_f firms adopt strategy f and μ_b firms opt for diversified supply chains, aggregate expected profits (net of any subsidies or taxes received or paid by firms in recognition of their supply chain choices) amount to $\sum_{j=h,f,b} \mu_j \Pi_j (\boldsymbol{\mu})$, where $\boldsymbol{\mu}$ is the vector, (μ_h, μ_f, μ_b). Consumer surplus in state J is given by $\frac{1}{\varepsilon-1} P^J (\boldsymbol{\mu})^{1-\varepsilon}$ for $J \in \{H, F, B\}$. In the event that both countries are hit with supply disruptions, which happens with probability $(1-\gamma_H)(1-\gamma_F)$, consumption of all differentiated products is zero ($X = 0$) and so consumer surplus vanishes. Therefore,

$$W(\boldsymbol{\mu}) = \bar{Y} + \sum_{J=H,F,B} \delta^J T^J (\boldsymbol{\mu}) + \sum_{j=h,f,b} \mu_j \Pi_j (\boldsymbol{\mu}) + \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J P^J (\boldsymbol{\mu})^{1-\varepsilon} \ , \tag{8}$$

where $\bar{Y}$ is the (fixed) labor income from producing the numeraire good, $T^J(\mu)$ is tax revenues collected by the government and rebated to households in state J (possibly zero or negative) beyond what is paid to or collected from firms in connection with their supply chain choices, and δ^J is the *ex ante* probability of state J, i.e., $\delta^H = \gamma_H (1-\gamma_F)$, $\delta^F = \gamma_F (1-\gamma_H)$, and $\delta^B = \gamma_H \gamma_F$.

3 The Market Equilibrium

In this section, we outline the conditions for market equilibrium in the absence of government intervention and describe how the resulting supply chains depend on the size of the fixed cost of forming a relationship. In the single time period that we consider, firms first choose their organizational strategies, h, f, or b. Then, the state of nature is realized, H, F, B, or N. The aggregate and idiosyncratic shocks determine which firms can operate. Those that are in a position to do so purchase their inputs from their minimum cost supplier. Given a firm's variable cost and the aggregator A^J for state J, equation (6) dictates its profit-maximizing price. At the same time, the collection of optimal pricing decisions determines the aggregator A^J for state J. Equation (7) delivers operating profits for any firm active in state J. We can use the operating profits in the various states to calculate a firm's expected profit under each entry strategy, which is a function of the measure of other firms that adopt each strategy. In equilibrium, all strategies that are chosen by a positive measure of firms must yield equal expected profits and such profits must be at least as large as what could be attained with strategies that are not pursued by any firms.

Market shares depend on firms' chosen prices relative to the aggregator A^J for state J. These shares must sum to one in every state. In state $J \in \{H, F\}$, we have

$$n^J (\boldsymbol{\mu}) \, s \left[z^J (\boldsymbol{\mu}) \right] = 1, \ J \in \{H, F\} \tag{9}$$

where $z^J(\boldsymbol{\mu}) = \frac{\sigma[z^J(\boldsymbol{\mu})]}{\sigma[z^J(\boldsymbol{\mu})]-1}\frac{q_J}{A^J(\boldsymbol{\mu})}$ is the relative price realized by every firm in state J, for $J \in \{H, F\}$, $n^H(\boldsymbol{\mu}) = \rho(\mu_h + \mu_b)$ and $n^F(\boldsymbol{\mu}) = \rho(\mu_f + \mu_b)$. In state B, firms producing at cost q_F achieve a relative price $z^{B,F}(\boldsymbol{\mu}) = \frac{\sigma[z^{B,F}(\boldsymbol{\mu})]}{\sigma z^{B,F}(\boldsymbol{\mu})-1}\frac{q_F}{A^B(\boldsymbol{\mu})}$ while those producing at q_H find $z^{B,H}(\boldsymbol{\mu}) = \frac{\sigma[z^{B,H}(\boldsymbol{\mu})]}{\sigma[z^{B,H}(\boldsymbol{\mu})]-1}\frac{q_H}{A^B(\boldsymbol{\mu})}$. Then we have

$$\frac{z^{B,F}(\boldsymbol{\mu})}{z^{B,H}(\boldsymbol{\mu})} = \frac{\frac{\sigma[z^{B,F}(\boldsymbol{\mu})]}{\sigma[z^{B,F}(\boldsymbol{\mu})]-1}}{\frac{\sigma[z^{B,H}(\boldsymbol{\mu})]}{\sigma[z^{B,H}(\boldsymbol{\mu})]-1}}\frac{q_F}{q_H}, \tag{10}$$

while (4) implies that

$$n^{B,H}(\boldsymbol{\mu})\, s\left[z^{B,H}(\boldsymbol{\mu})\right] + n^{B,F}(\boldsymbol{\mu})\, s\left[z^{B,F}(\boldsymbol{\mu})\right] = 1, \tag{11}$$

where $n^{B,F} = (\mu_f + \mu_b)\rho$ and $n^{B,H} = \mu_h\rho + \mu_b\rho(1-\rho)$. Now we can use (9) for $J \in \{H, F\}$, together with (10) and (11) to solve for the four relative prices as functions of $\boldsymbol{\mu}$.

Operating profits for firms that do not suffer supply disruptions are given by

$$\pi^J(\boldsymbol{\mu}) = \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]}P^J(\boldsymbol{\mu})^{1-\varepsilon}, \text{ for } J \in \{H, F\},$$

and

$$\pi^{B,i}(\boldsymbol{\mu}) = \frac{s\left[z^{B,i}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,i}(\boldsymbol{\mu})\right]}P^B(\boldsymbol{\mu})^{1-\varepsilon}, \ i \in \{H, F\},$$

where the expressions for the price indexes are given in (23) and (34) in the appendix.

Finally, we calculate the expected profits a firm can achieve by adopting each available strategy,

$$\Pi_h(\boldsymbol{\mu}) = \delta^H\pi^H(\boldsymbol{\mu})\rho + \delta^B\pi^{B,H}(\boldsymbol{\mu})\rho - k\ , \tag{12}$$

$$\Pi_f(\boldsymbol{\mu}) = \delta^F\pi^F(\boldsymbol{\mu})\rho + \delta^B\pi^{B,F}(\boldsymbol{\mu})\rho - k\ , \tag{13}$$

and

$$\Pi_b(\boldsymbol{\mu}) = \delta^H\pi^H(\boldsymbol{\mu})\rho + \delta^F\pi^F(\boldsymbol{\mu})\rho + \delta^B\pi^{B,F}(\boldsymbol{\mu})\rho + \delta^B\pi^{B,H}(\boldsymbol{\mu})\rho(1-\rho) - 2k\ . \tag{14}$$

Each firm chooses a strategy that is among those that maximize expected profits. In equilibrium, $\boldsymbol{\mu}$ must be consistent with optimization by all firms.

Figure 1 shows the fraction of firms that choose each organizational form for different values of k.[12] For $k < k_1$, all firms find it worthwhile to invest in resilience, so $\mu_b = 1$ and $\mu_h = \mu_f = 0$. Next comes a range of fixed costs for which some firms are diversified, and others form relationships in a single country, with all sole-source suppliers located either in country H or in country F. The figure depicts a case with $q_H \approx q_F$ and $\gamma_H > \gamma_F$. In such circumstances, there exists a k_2 such

[12] Details behind Figure 1 and formal arguments about the sorting pattern may be found in the appendix of our working paper, Grossman et al. (2021).

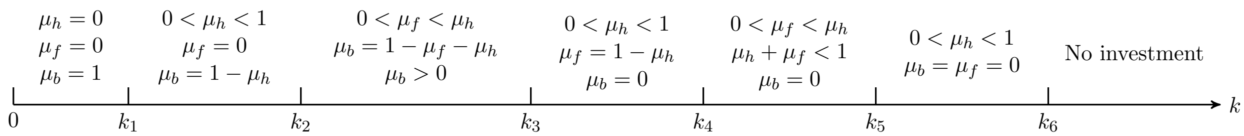

Figure 1: Supply Chain Outcomes

that for $k \in (k_1, k_2)$, $\mu_h = 1 - \mu_b > 0$ and $\mu_f = 0$. Alternatively if $q_F < q_H$ and $\gamma_H \approx \gamma_F$, then there exists a k_2 such that for $k \in (k_1, k_2)$, $\mu_f = 1 - \mu_b > 0$ and $\mu_h = 0$.[13] Next, there exists a range of fixed costs $k \in (k_2, k_3)$ for which each of the three strategies is deployed in equilibrium by some positive number of firms. As k rises further, diversification becomes unprofitable for all firms ($\mu_b = 0$), then sole sourcing becomes unprofitable in one of the two countries, and finally entry of any sort becomes unprofitable.[14]

In what follows, we will be most interested in circumstances that give rise to the use of all available strategies in equilibrium, i.e., $\mu_h > 0$, $\mu_f > 0$, and $\mu_b > 0$. Therefore, we focus on cases with $k \in (k_2, k_3)$.

4 The Unconstrained Social Optimum

Do private incentives for firms to invest in safe and resilient supply relationships align with social incentives? The answer to this question may depend on the set of policy instruments that the government has at its disposal to influence resource allocation. As is well known, the markup pricing reflected in equation (6) creates a wedge between social and private incentives to consume differentiated products relative to the homogeneous good, because consumers face a price in excess of marginal cost for the former, but a price equal to marginal cost for the latter. As in other contexts, this distortion can be eliminated, in principle, by an optimal set of consumption subsidies. In practice, such subsidies are difficult to implement and rarely observed; in our setting with non-constant markups, the requisite subsidies must vary with both the state of nature (H, F, or B) and with the source of the inputs embodied in the final good.[15] Nonetheless, it is instructive to begin our analysis under the assumption that optimal subsidies are feasible, to focus squarely on the wedges between private and social incentives for supply chain formation. In this section, we study the unconstrained (or "first-best") planner's problem, leaving the more realistic, constrained (or "second-best") problem for the next section. By assuming away the distortions caused by markup

[13] If $q_F < q_H$ and $\gamma_H > \gamma_F$, then we might have a range of fixed costs for which $\mu_H > 0 = \mu_F$ or with $\mu_F > 0 = \mu_H$, depending on whether the prospective cost savings in the foreign country outweigh the relatively greater reliability of the home suppliers.

[14] For $k \in (k_3, k_4)$, all firms form a single relationship either in H or in F, so that $\mu_h + \mu_f = 1$, whereas for $k \in (k_4, k_5)$ some firms choose not to invest at all, so that $\mu_h + \mu_f < 1$.

[15] In state B, some final producers source from suppliers in the foreign country at unit cost q_F while others source from the home country at the higher cost q_H. The subsidies needed to ensure that consumers see relative prices equal to relative marginal costs will vary, therefore, with the sourcing of the inputs, despite the fact that all final goods enter demand symmetrically.

pricing, we are able to develop intuition for whether and when the incentives firms face to invest in safety and resilience are excessive, insufficient or appropriate.

With optimal consumption subsidies that eliminate the wedges between consumer prices and marginal costs, we can write a simpler expression for welfare. In the appendix, we derive (41), which we record here as

$$\tilde{W}(\boldsymbol{\mu}) = \bar{Y} + \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J \tilde{P}^J(\boldsymbol{\mu})^{1-\varepsilon} - k(\mu_H + \mu_f + 2\mu_b), \tag{15}$$

where the tildes indicate relationships that apply with the optimal consumption subsidies in place. Notice that the terms for tax revenues, $T^J(\boldsymbol{\mu})$ and for the aggregate expected profits, $\sum_j \mu_j \Pi_j(\boldsymbol{\mu})$ in (8) are missing from (15). Intuitively, the consumption subsidies match the firms' markups, which leaves the cost of the consumption subsidy program as an exact offset to firms' operating profits. What remains from (8) is labor income, expected consumer surplus, and the total fixed costs of the supply links.

We focus on $k \in (k_2, k_3)$ in Figure 1, so that all three strategies are deployed in equilibrium and $\mu_b = 1 - \mu_h - \mu_f$. To characterize the optimal supply chain policies, we define the wedge between private and social incentives to pursue strategy j relative to strategy b when the existing mix of strategies is $\boldsymbol{\mu}$ as

$$\tilde{w}_j(\boldsymbol{\mu}) := \left[\tilde{\Pi}_j(\boldsymbol{\mu}) - \tilde{\Pi}_b(\boldsymbol{\mu})\right] - \frac{d\tilde{W}(\boldsymbol{\mu})}{d\mu_j},\ j \in \{h, f\}, \tag{16}$$

where $\tilde{\Pi}_j$ is the expected profit from strategy j with optimal consumption subsidies in place, $\tilde{\Pi}_b$ is the expected profit from strategy b with optimal subsidies in place, and

$$\frac{d\tilde{W}(\boldsymbol{\mu})}{d\mu_j} := \frac{\partial \tilde{W}(\mu_h, \mu_f, \mu_b)}{\partial \mu_j} - \frac{\partial \tilde{W}(\mu_h, \mu_f, \mu_b)}{\partial \mu_b},\ j \in \{h, f\}$$

is the marginal change in welfare from a small change in μ_j at the expense of μ_b; i.e., $d\mu_j = -d\mu_b > 0$.[16]

At an (interior) first-best allocation $\boldsymbol{\mu}^o$, the first-order conditions for welfare maximization require $d\tilde{W}_j(\boldsymbol{\mu}^o)/d\mu_j = 0$ for $j \in \{h, f\}$. Therefore, the first best can be achieved by a set of subsidies that satisfy

$$\tilde{w}_j(\boldsymbol{\mu}^o) = \tilde{\Pi}_j(\boldsymbol{\mu}^o) - \tilde{\Pi}_b(\boldsymbol{\mu}^o) = \varphi_b - \varphi_j,\ j \in \{h, f\}, \tag{17}$$

where φ_j, $j = h, f$, is a subsidy (possibly negative) paid unconditionally to any firm that pursues strategy j and φ_b is a subsidy (possibly negative) paid unconditionally to a firm that diversifies its sourcing options. The optimal subsidies/taxes offset the wedges that remain at the optimal allocation.

We have given the social planner three policy instruments to counteract two wedges. Clearly,

[16] We henceforth use the notation of total derivaties, $dG(\boldsymbol{\mu})/d\mu_j$, for $j \in \{h, f\}$, to denote the marginal change in the function $G(\cdot)$ with respect to μ_j, taking into account that $\mu_b = 1 - \mu_h - \mu_f$.

she has a degree of freedom in her policy choices. The first best can be achieved with a continuum of combinations of subsidies/taxes, including ones that eschew the use of one instrument entirely.

In the appendix, we compute the two wedges, (47) and (48), and find that

$$\tilde{w}_f^o := \tilde{w}_f(\boldsymbol{\mu}^o) = \delta^H \Phi\left[z\left(\mu_f^o\right)\right] \tilde{P}^H(\boldsymbol{\mu}^o)^{1-\varepsilon} \rho + \delta^B \Phi\left[z^{B,H}(\boldsymbol{\mu}^o)\right] \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \rho(1-\rho) \tag{18}$$

and

$$\begin{aligned}\tilde{w}_h^o := \tilde{w}_h(\boldsymbol{\mu}^o) = \delta^F \Phi\left[z\left(\mu_h^o\right)\right] \tilde{P}^F(\boldsymbol{\mu}^o)^{1-\varepsilon} \rho + \delta^B \Phi\left[z^{B,H}(\boldsymbol{\mu}^o)\right] \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \rho(1-\rho) \\ +\delta^B\left\{\Phi\left[z^{B,F}(\boldsymbol{\mu}^o)\right] - \Phi\left[z^{B,H}(\boldsymbol{\mu}^o)\right]\right\} \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \rho\ ,\end{aligned} \tag{19}$$

where

$$\Phi(z) := \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta - \frac{s(z)}{\sigma(z)-1}.$$

Assumption 2(i) specifies $\sigma'(z) > 0$ for all $z \in (0, \bar{z})$ (i.e., Marshall's Second Law of Demand) or else σ constant for all $z \in (0, \bar{z})$ as a limiting case (i.e., Symmetric CES preferences). Taking the latter first, a constant elasticity of substitution arises when $s(z) = \alpha z^{1-\sigma}$. Then it is straightforward to see that $\Phi(z) = 0$ for all $z \in (0, \bar{z})$. This in turn implies $w_f^o = w_h^o = 0$; both wedges are zero at the unconstrained optimum. In contrast, when the elasticity of substitution rises with the relative price, we show in (27) in the appendix that $\Phi(z) < 0$ for all $z \in (0, \bar{z})$. Then (18) implies immediately that $w_f^o < 0$, since both terms on the right-hand side are negative. As for strategy h, we have that $\tilde{z}^{B,H}(\boldsymbol{\mu}^o) > \tilde{z}^{B,F}(\boldsymbol{\mu}^o)$, because efficient relative prices are equal to relative marginal costs, and $q_H > q_F$. Together with $\Phi'(z) > 0$, this implies $w_h^o < 0$ as well.

The negative wedges imply that, at the unconstrained optimum, firms have excessive incentives for diversification; that is, converting a firm with an exclusive relationship in either country to one that is diversified will reduce aggregate welfare. To interpret this finding, note that firms' investments in supply chains determine the number of differentiated products available in each state of the world, as well as (with MSLD) their markups and prices. An increase in μ_b at the expense of μ_f, for example, means that more products are available in state H, when the foreign supply is disrupted, because diversified firms can source from the home country, but f firms have no suppliers there. It also implies that more products are available in state B—when there are no country-wide supply disturbances.

When a firm chooses its investment strategy and thereby affects the number of varieties available in different states, it conveys two externalities. On the one hand, consumers love variety and they reap consumer surplus from greater availability at a given price. Firms do not take account of this positive effect of their product's availability on consumer surplus when forming their supply chains. On the other hand, more variety spells less demand and less profits for any particular product at given prices. Firms do not take account of this adverse effect of their product's availability on the profits earned by others. The sign of the wedge at the optimal allocation reflects the relative sizes of these two countervailing externalities.

Now consider, for example, how a change in the number of products n^H available in state H affects the gap between the price index, P^H, and the demand aggregator, A^H. The former fully captures the effect of product availability on consumer surplus whereas the effect on aggregate profits also depends on the latter. Using (5), (6) and (9) we calculate

$$\frac{1}{P^H}\frac{dP^H}{dn^H} - \frac{1}{A^H}\frac{dA^H}{dn^H} = \frac{s\left(z^H\right)}{\sigma\left(z^H\right)-1} - \int_{z^H}^{\bar{z}} \frac{s\left(\zeta\right)}{\zeta} d\zeta = -\Phi\left(z^H\right).$$

When preferences satisfy MSLD, an extra variety in state H (or in state B) reduces the price index for that state by proportionately less than it does the demand aggregator. Thus, the positive consumer-surplus externality from added availability in states H and B falls short of the negative business-stealing externality. In such circumstances, the private incentives for resilience exceed the social incentives.[17]

Similar forces are at work with respect to a firm's choice between strategy h and strategy b. An increase in μ_b at the expense of μ_h means greater availability in state F (when the home suppliers are disrupted) and in state B (when diversified firms stand a better chance of avoiding the disruption from a idiosyncratic shocks). The first two (negative) terms on the right-hand side of (19) reflect the excessive private incentive for product availability in these states relative to the social value of more varieties and are analogous to the similar terms in (18). In fact, an increase in μ_b at the expense of μ_h has exactly the same effect on the number of varieties available in state B as does a similar increase in μ_b at the expense of μ_f.

However, an increase in μ_b at the expense of μ_h has a further effect on the wedge w_h, as represented by the third term on the right-hand side of (19). When μ_b rises and μ_h falls, the extra products that become available in state B are low-cost goods, whereas when μ_b rises and μ_f falls, the marginal products are high-cost goods. Extra low-cost goods take a greater toll on the profits of competitors than do extra high-cost goods, which adds the additional negative term to the wedge w_h^o.

We turn now to the policies that the planner can introduce, alongside the optimal, state-and-product-contingent consumption subsidies, to implement the first best. Let us begin with the limiting case of symmetric, CES preferences. With $w_f^o = w_h^o = 0$, the optimum can be achieved without any intervention in supply chain formation; i.e., $\varphi_h = \varphi_f = \varphi_b = 0$. CES preferences are unique in the set of HSA preferences inasmuch as the price index is proportional to the demand aggregator. Then the external effects of a product's availability on consumers and competitors are equal in magnitude and opposite in sign. Once optimal consumption subsidies are in place to counter the distortion created by markup pricing, there is no need for further government policy to influence the number of products available in any state.

Next consider the special case of symmetric translog preferences, for which MSLD applies. In this case, with $w_f^o < 0$ and $w_h^o < 0$, the government must discourage investments in resilience

[17] This result echoes that in Matsuyama and Uschev (2020a) that there is excessive entry under MSLD in a one-sector model of monopolistic competition and no supply shocks.

and encourage firms to develop exclusive sourcing arrangements. But we can say more. Using $s(z) = -\theta \log z$, we find $\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta = \frac{1}{2} \frac{s(z)}{\sigma(z)-1}$ for all $z \in (0,1)$. Then using the planner's first-order conditions for the optimal choice of μ_h and μ_f, along with (32)-(34) in the appendix, we find that $w_f^o = w_h^o = -k$; see Appendix Lemma 3. The two wedges are identical to one another for all values of the cost parameters (q_H, q_F) and all values of the risk parameters $(\gamma_H, \gamma_F, \rho)$, and they are equal in absolute value to the fixed cost of forming a supply relationship. The planner can achieve the first best by combining the optimal consumption subsidies with a tax on diversification; $\varphi_b = -k$, with $\varphi_h = \varphi_f = 0$. Alternatively, she can leave diversified firms to face the private cost of their supply chains ($\varphi_b = 0$), while subsidizing firms that form exclusive supply relationships to the full extent of their investment costs ($\varphi_h = \varphi_f = k$). In either case, she has no reason to favor onshore investments relative to offshore investments.

These surprising results reflect a special property of symmetric translog preferences, namely that the ratio of $\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta$ to $\frac{s(z)}{\sigma(z)-1}$ is independent of price and always equal to one half. Since the consumer surplus loss from removing a variety in some state is proportional to $\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta$, while the loss in operating profits for the firm that does not produce its variety (with the optimal consumption subsidy in place) is proportional to $\frac{s(z)}{\sigma(z)-1}$, this property of the translog preferences implies that the consumer surplus loss from switching a firm from having two suppliers to one is one half the loss in operating profits. The wedge w_j^o is equal to the difference in expected profits, $\left[\tilde{\Pi}_j(\boldsymbol{\mu}) - \tilde{\Pi}_b(\boldsymbol{\mu})\right]$ per (17), which in turn is equal to the fixed cost of a relationship minus the loss in operating profits. Finally, the first-order condition for maximizing $\tilde{W}(\boldsymbol{\mu})$ in (15) dictates that the marginal loss of consumer surplus from switching a firm from strategy b to strategy j should match the cost of an extra supplier, k.

With more general HSA preferences, the incentives created by optimal supply chain policy are not neutral with respect to the location of firms' input suppliers. The optimal policies favor onshore relationships relative to offshore relationships if $|w_h^o| > |w_f^o|$ and offshore relationships relative to onshore relationships if the ranking of the two wedges is reversed.

To shed further light on the desired national bias in first-best supply chain policy, we examine the limiting case when production costs are nearly the same, $q_H \searrow q_F$. Then we find in (52) in the appendix that

$$|w_h^o| - |w_f^o| \propto \Psi\left[z^H(\boldsymbol{\mu}^o)\right] - \Psi\left[z^F(\boldsymbol{\mu}^o)\right]$$

where $\Psi(z) := \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta / \left[\frac{s(z)}{\sigma(z)-1}\right]$. In the appendix, we also show that $z^H(\boldsymbol{\mu}^o) < z(\boldsymbol{\mu}^o)$. It follows that, with nearly equal costs, the government should encourage the less risky investments at home relative to the more risky investments abroad if and only if $\Psi(z)$ is a decreasing function.

When $q_H \gg q_F$, this simple reasoning does not apply, because the planner's preference for home sourcing on safety grounds is counteracted by her preference for foreign sourcing on cost grounds, so that $\mu_h^o \lesseqgtr \mu_f^o$. Moreover, with unequal costs, an increase μ_b at the expense of μ_h makes a greater contribution to consumer surplus in state B while taking a greater toll on rivals' profits than does an increase in μ_b at the expense of μ_f; although both increase product availability in state B by similar amounts, the former spells greater availability of low-cost products, whereas the latter generates greater availability of high-cost products.

Equations (18) and (19) imply

$$\begin{aligned} |w_h^o| - \left|w_f^o\right| &= \delta^H \Phi\left[z^H\left(\boldsymbol{\mu}^o\right)\right] \tilde{P}^H\left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \rho - \delta^F \Phi\left[z^F\left(\boldsymbol{\mu}^o\right)\right] \tilde{P}^F\left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \rho \\ &\quad + \delta^B \left\{\Phi\left[z^{B,H}\left(\boldsymbol{\mu}^o\right)\right] - \Phi\left[z^{B,F}\left(\boldsymbol{\mu}^o\right)\right]\right\} \tilde{P}^B\left(\boldsymbol{\mu}^o\right)^{1-\varepsilon} \rho. \end{aligned}$$

The difference between the first two terms—i.e., whether the gap between social and private incentives is greater in state H or state F—depends in a subtle way on whether $\mu_h^o > \mu_f^o$ and whether $\Psi(z)$ is increasing or decreasing. The third term contributes to a positive wedge that favors onshoring if $\Psi'(z) > 0$ but to a negative wedge that favors offshoring if $\Psi'(z) < 0$.

We summarize our findings about first-best supply-chain policy in

Proposition 1 *Under Assumptions 1 and 2, the unconstrained planner uses consumption subsidies to undo the markup distortion for each good in each state of nature. With symmetric CES preferences, a hands-off policy with respect to supply chain formation ($\varphi_h = \varphi_f = \varphi_b = 0$) achieves the first best. Under MSLD, the planner encourages single sourcing relationships relative to diversification. With symmetric translog preferences, the planner can achieve the first best with a tax on diversification of size k. More generally, if cost differences are small ($q_H \searrow q_F$), the planner encourages onshore sourcing relative to offshore sourcing if $\Psi'(z) > 0$ for all $z \in (0, \bar{z})$ and encourages offshore sourcing relative to onshore sourcing if $\Psi'(z) < 0$ for all $z \in (0, \bar{z})$. For larger cost differences, the national bias in optimal sourcing policy hinges not only on the sign of $\Psi'(z)$, but also on the magnitudes of the cross-country cost and risk differences.*

5 The Constrained Social Optimum

In the last section, we characterized the first-best allocation of resources when firms in a monopolistically competitive industry face potential supply chain disruptions. We noted that attainment of the first best requires not only that government use policies to offset distortions in firms' incentives for forming supply relationships, but also a policy to counter the distortion created by monopoly pricing of differentiated products alongside the competitive pricing of goods elsewhere in the economy. As we discussed, the requisite consumption subsidies are rarely implemented in practice. In our context, not only would they need to be adjusted in response to realized disruptions, but they would also need to vary across otherwise symmetric products that differ only in the sourcing of their critical inputs. Nonetheless, by allowing for optimal consumption subsidies, we were able to lay bare the wedges between private and social incentives for supply diversification and for onshoring versus offshoring.

In this section, we consider the second-best problem that confronts a welfare-maximizing government that lacks the ability to implement state-contingent and sourcing-contingent consumption subsidies. We grant the policy maker only taxes or subsidies to encourage or discourage supply

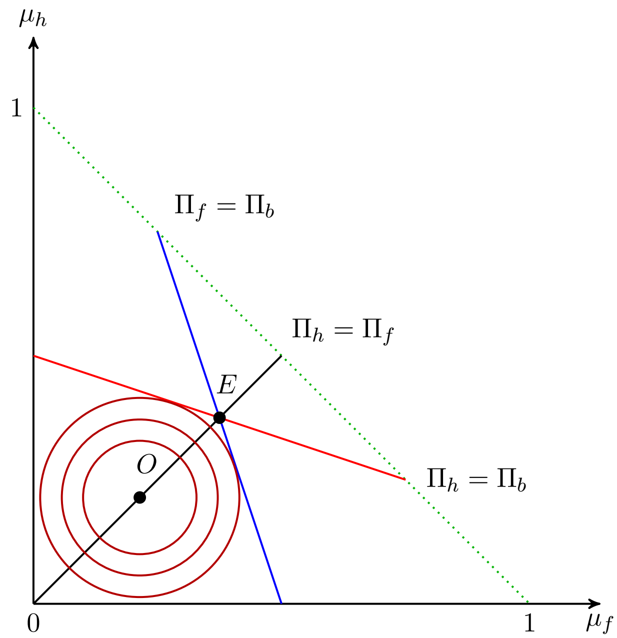

Figure 2: Equilibrium and Constrained Optimum for the Symmetric Case

Note: Figure drawn for $k \in (k_2, k_3)$, so that $\mu_f > 0$, $\mu_h > 0$, and $\mu_b = 1 - \mu_h - \mu_f$.

chain resilience and to influence whether sourcing partnerships are formed at home or abroad. As with the unconstrained optimum, the constrained social optimum can be achieved by a continuum of combinations of subsidies or taxes for the formation of home relationships, foreign relationships, and multiple relationships and, indeed, any two of these three instruments will suffice.

Let us begin with the limiting case in which the home and foreign countries are symmetric in terms of input costs and disruption risks; i.e., $q_H \searrow q_F$ and $\gamma_H \searrow \gamma_F$. In Figure 2, we illustrate a *laissez-faire* equilibrium at E for a typical case in which all three strategies are employed by positive measures of firms; i.e., when fixed cost of forming relationships fall in the range (k_2, k_3). The figure shows μ_h and μ_f on the vertical and horizontal axes, respectively. Since k is in the range where all firms invest in at least one supply relationship, Figure 2 depicts the plane in (μ_f, μ_h, μ_b) space along which $\mu_b = 1 - \mu_f - \mu_h$.

The curve labeled $\Pi_h = \Pi_f$ represents combinations of μ_h and μ_f such that a strategy of forming a single supply relationship at home yields the same expected profit as that of forming a single supply relationship abroad. With similar production costs, both strategies yield the same profits in state B, when sourcing from both countries is viable for any firm that avoids an idiosyncratic supply shock. Profits in state H for a firm that adopts strategy h are declining in the number of firms that are active in that state, while profits in state F for a firm that adopts strategy f are similarly declining in the number of firms that are active in that state. Meanwhile, firms that adopt strategy h earn no profits in state F and firms that adopt strategy f earn no profits in state H. It follows that equal expected profitability of the two strategies requires similar numbers of active firms in state H and state F, which in turn requires $\mu_h = \mu_f$. Thus, we represent the $\Pi_h = \Pi_f$ curve by a 45° ray from the origin.

The downward sloping curve labelled $\Pi_h = \Pi_b$ shows combinations of μ_h and μ_f (with $\mu_b = 1 - \mu_f - \mu_h$) for which investing in a single relationship at home yields the same expected profits as a strategy of diversification. Note that, with $\gamma^H = \gamma^F = \gamma$, (12) and (14) yield the equation for this curve, namely

$$\gamma (1-\gamma) \pi^F (\boldsymbol{\mu}) \rho + \gamma^2 \pi^B (\boldsymbol{\mu}) \rho (1-\rho) = k. \tag{20}$$

The downward slope of the curve can be understood as follows. Starting from a point on the curve, say E, suppose we move vertically upward. This shift corresponds to a rise in μ_h and a decline in μ_b of similar magnitudes, with μ_f held constant. The change in composition of strategies does not affect the total number of firms active in state H, but it decreases the numbers of firms active in states F and B. Thus, π^F and π^B rise, leaving a (positive) gap between Π_b and Π_h for points vertically above E.[18] Now consider a fall in μ_f accompanied by an offsetting rise in μ_b at a given μ_h, i.e., a horizontal movement to the left. This change in the composition of firms intensifies competition in state H, which contributes to lower expected profits for both h firms and b firms. Since both types earn operating profits of π^H with the same probability ρ in state H, the fall in π^H does not figure in the comparison between the two. Meanwhile, in state F, the offsetting changes in μ_f and μ_b leave the intensity of competition untouched and with it the operating profits π^F for any firm that is active in this state. Finally, in state B, the number of active firms rises, because a given firm is more likely to produce in this state if it is diversified than if it has only a single potential supplier. The intensification of competition in state B reduces π^B, which depresses expected profits more for diversified firms than for those that invest only in a domestic supplier, because the former firms are more likely to survive in this state. Thus, a decrease in μ_f offset by an increase in μ_b reduces Π_b relative to Π_h. It follows that a decrease in μ_f is needed to offset the effects of an increase in μ_h if strategies h and b are to be equally profitable. We note further that the $\Pi_h = \Pi_b$ curve must have a slope less than one in absolute value.[19]

The curve labelled $\Pi_f = \Pi_b$ shows combinations of μ_h and μ_f for which investing in a single relationship abroad yields the same expected profits as a strategy of diversification. It too slopes downward, for analogous reasons, and the curve must have a slope greater than one in absolute value. Finally, the three curves are shown to intersect at E, where all three strategies are equally profitable, as befits an equilibrium in which positive numbers of firms select each option.

The figure also illustrates a constrained optimum at O. The constrained optimum maximizes

[18] The fall in competition in state F serves to increase expected profits of diversified firms, but does not affect the expected profits of firms that can only source at home. The fall in competition in state B raises the profitability of an active h firm and an active b firm by similar amounts, but the diversified firms have a better chance of avoiding supply disruptions, so they reap a bigger boost to expected profits in this state as well.

[19] The effect on expected profits of a diversified firm relative to a home-only firm conditional on state B are equal and opposite for a given increase in μ_h and comparable decrease in μ_f. But the increase in μ_h (and accompanying decrease in μ_b) gives an added boost to the relative profitability of diversification, because it raises the expected profits for a b firm if state F arises. See the appendix to our working paper, Grossman et al. (2021), for further details.

W over the choice of $\boldsymbol{\mu}$ in the presence of monopoly pricing of differentiated products. In the appendix, we show that the first-order conditions for a constrained maximum are satisfied with an appropriate choice of $\mu_h = \mu_f$ for any HSA preferences. The figure depicts some iso-welfare loci for successively lower levels of expected welfare as we move away from O. Notice that they are symmetric about the 45-degree line, thanks to the symmetry across countries.

It should be clear that, in a symmetric case in which O falls on the 45° line, the constrained optimum can be achieved with a tax or subsidy on diversification alone, with $\varphi_f = \varphi_h = 0$. Such a policy shifts the equilibrium along the $\Pi_h = \Pi_f$ curve and thereby preserves the equality between μ_h and μ_f. What remains to be addressed is whether the government should encourage diversification with a subsidy for firms that form multiple relationships ($\varphi_b > 0$) or whether it should discourage diversification with a tax ($\varphi_b < 0$) on such firms. This amounts to the same question as to whether point O lies to the southwest of E along $\Pi_h = \Pi_f$ or whether it lies instead to the northeast of E.

We can answer these questions formally using methods similar to the ones we applied in Section 4. We begin with the planner's objective in (8), with $T^J \equiv 0$ for all $J \in \{H, F, B\}$ in the absence of consumption subsidies.[20] The wedge between social and private incentives for diversification is given by

$$w_j^* := \Pi_j\left(\boldsymbol{\mu}^*\right) - \Pi_b\left(\boldsymbol{\mu}^*\right) - \frac{dW\left(\boldsymbol{\mu}^*\right)}{d\mu_j},\ j \in \{h, f\},$$

where $\boldsymbol{\mu}^*$ represents the allocation in the constrained optimum and recall that $dG\left(\boldsymbol{\mu}\right)/d\mu_j$ denotes the variation in any function $G\left(\boldsymbol{\mu}\right)$ for $d\mu_j = -d\mu_b > 0$. The second-best allocation satisfies the first-order conditions,

$$\frac{dW\left(\boldsymbol{\mu}^*\right)}{d\mu_j} = \Pi_i\left(\boldsymbol{\mu}^*\right) - \Pi_b\left(\boldsymbol{\mu}^*\right) + \sum_{i=h,f,b} \mu_i \frac{d\Pi_i\left(\boldsymbol{\mu}^*\right)}{d\mu_j}$$
$$+ \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J \frac{d\left[P^J\left(\boldsymbol{\mu}^*\right)^{1-\varepsilon}\right]}{d\mu_j} = 0 \text{ for } j \in \{h, f\}.$$

It follows that

$$w_j^* = -\sum_{i=h,f,b} \mu_i \frac{d\Pi_i\left(\boldsymbol{\mu}^*\right)}{d\mu_j} - \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J \frac{d\left[P^J\left(\boldsymbol{\mu}^*\right)^{1-\varepsilon}\right]}{d\mu_j} \text{ for } j \in \{h, f\}. \tag{21}$$

The first term on the right-hand side of (21) represents the business-stealing externality; i.e., the change in other firms' profits that results from shifting a marginal firm from diversified sourcing to sole sourcing in country i. The second term represents the consumer-surplus externality; i.e., the change in consumer-surplus that results from reduced product availability and higher prices in the three states. The difference from the analogous expressions in Section 4 reflects the fact that the constrained policy maker needs to take account not only of the *direct* effects on profits

[20]Recall that the cost of subsidies or taxes for supply chain formation are exactly offset by increases or decreases in firms' expectedrepresents the allo profits.

and consumer surplus of changing the numbers of firms in each state (holding prices constant), but also the *indirect* effects on profits and consumer surplus that come from marginal adjustments in the markups. The unconstrained planner can ignore these latter effects when deciding $\boldsymbol{\mu}^o$, because the choice of optimal consumption subsidies ensures that the induced changes in markups have a negligible effect on aggregate utility.

In the appendix, we provide a general formula for the wedge w_i^* for an arbitrary share function $s(z)$ that satisfies Assumptions 1 and 2. Then we turn to the symmetric case depicted in Figure 2, where $w_h^* = w_f^* = w^*$. Point E lies above point O whenever $w^* > 0$ and below point O whenever $w^* < 0$.

The results for our two special cases of HSA preferences are instructive. First, with *symmetric CES preferences*, we find that $w^* > 0$ and thus $\varphi_b^* > 0$ for all $\sigma > 1$. Recall that with optimal consumption subsidies in place, the optimal policy has $\varphi_b^o = 0$, because the extra consumer surplus generated by adding firms in a given state exactly matches the loss in aggregate profits. Now, with consumption subsidies unavailable to the policy maker, the monopoly pricing of differentiated products generates too little consumption of these goods relative to the numeraire good in the *laissez-faire* equilibrium. A subsidy for diversification increases the number of available products in every state, which reduces P^J for all $J \in \{H, F, B\}$, even though prices of marketed products do not change. The fall in the price index in state J stimulates consumption of differentiated products in that state, thereby mitigating the consumption distortion.

Second, with *symmetric translog preferences*, we show that $w^* > 0$ if $\varepsilon > \frac{\theta\rho(2-\rho)[1+\theta\rho(2-\rho)]}{1+3\theta\rho(2-\rho)}$ and $w^* < 0$ if $\varepsilon < \frac{\theta\rho(2+\theta\rho)}{2(2+3\theta\rho)}$; see Lemma 7. Recall that a tax on diversification is needed to align social and private incentives for supply chain formation for any HSA preferences other than CES when a consumption subsidy is available to correct the distortion otherwise generated by markup pricing. In the absence of consumption subsidies, the tendency for firms to overinvest in resilience continues to figure in the policy maker's calculus, because the business-stealing externality is large relative to the consumer-surplus externality. However, the distortion arising from monopoly pricing points in the opposite direction; there is too little consumption of differentiated products relative to the numeraire good and a subsidy for diversification would boost consumption of these goods. When demand for differentiated goods is highly elastic, the distortion from monopoly pricing looms large and the planner's imperative to encourage consumption of these goods outweighs her concern about firms' excessive investments in resilience, much as with CES preferences. In contrast, when demand for differentiated products is not so elastic, the welfare effects of the consumption distortion are muted and the planner acts to dampen firms' excessive incentive to be present in the market.

Let us return now to the case in which input costs are lower abroad than at home ($q_H > q_F$) but foreign sourcing entails greater risk of disruption than home sourcing ($\gamma_H > \gamma_F$). Figure 3 depicts the *laissez-faire* equilibrium and the constrained optimum in such a setting for general HSA preferences. Again we consider fixed costs of sourcing relationships in the range that a positive measure of firms chooses each of the available investment strategies. For $k \in (k_2, k_3)$, there is no exit, so $\mu_b = 1 - \mu_h - \mu_f$. As in Figure 2, the curve $\Pi_h = \Pi_f$ depicts combinations of μ_h and μ_f

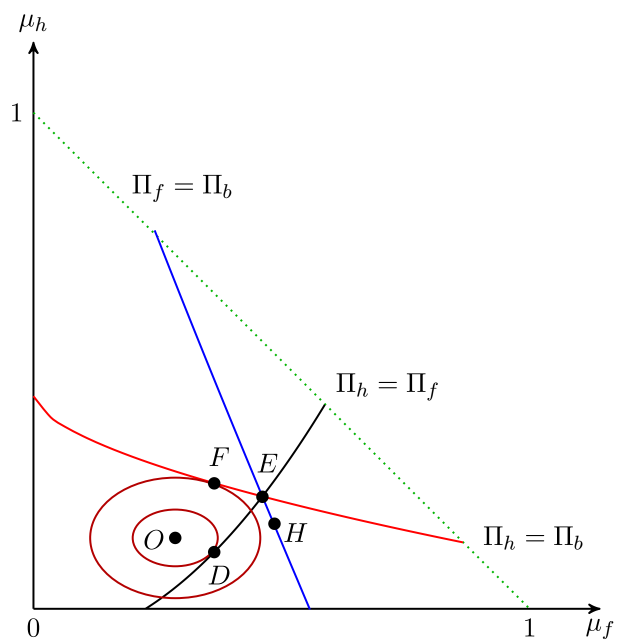

Figure 3: Equilibrium and Constrained Optimum for the Asymmetric Case

Note: Figure drawn for $k \in (k_2, k_3)$, so that $\mu_f > 0$, $\mu_h > 0$, and $\mu_b = 1 - \mu_h - \mu_f$.

(with $\mu_b = 1 - \mu_h - \mu_f$) that deliver equal expected profits to firms that form supply chains only in H or only in F. The curves labeled $\Pi_h = \Pi_b$ and $\Pi_f = \Pi_b$, respectively, show combinations of μ_h and μ_f such that investing in a single relationship in H or in F yields the same expected profits as investing in both. With all strategies in use, the *laissez-faire* equilibrium E lies at the intersection of the three curves.

We observe first that, for general HSA preferences, the constrained optimum, O, need not fall on any of the three equiprofitability curves. This means that, generically, the government cannot achieve the constrained optimum with a single policy instrument. For the case illustrated in Figure 3, a subsidy for diversification improves welfare relative to E, but since such a policy preserves $\Pi_h = \Pi_f$, such a policy can achieve at best the utility associated with the iso-welfare curve through point D. A tax to discourage sourcing abroad ($\varphi_f < 0$) shifts the equilibrium to the left along the $\Pi_h = \Pi_b$ curve, but at best can achieve the utility associated with the iso-welfare curve through point F. Finally, a tax on onshore relationships with a single partner ($\varphi_h < 0$) can be used to achieve point H. For the preferences and parameters depicted in the figure, the constrained optimum could be achieved with a combination of a subsidy for diversification and a tax on sole-sourcing offshore or with a subsidy to diversification and a subsidy for sole-sourcing at home. The subsidy for diversification would shift the equilibrium to some point along $\Pi_h = \Pi_f$ to the southwest of E, while either a tax on offshoring or a subsidy to onshoring would shift the equilibrium to the equilibrium from there up and to the left.

Although the constrained optimum can be characterized for particular preferences and parameters, in general the wedges w_h^* and w_f^* that dictate the second-best policy combinations can take any sign and a range of relative magnitudes. The reason for this reflects the complexity of the planner's constrained maximization problem. With policies that alter μ_h, μ_f and μ_b, the government can change the number of products available in the three possible states of the world in specific ways. In particular, we know that $n^H = \rho(\mu_h + \mu_b)$, $n^F = \rho(\mu_f + \mu_b)$ and $n^B = \rho + \rho(1-\rho)\mu_b$. So a combination of policies that encourages greater diversification at the expense of sole-sourcing offshore, for example, generates an increase in the number of products available in state H by $d\mu_b$ varieties and increases the number of products available in state B by$\rho(1-\rho)\,d\mu_b$ varieties, while leaving the number available in state F unchanged. But the size of the consumption distortion from monopoly pricing varies across different states of the world, because the markups depend on the number of products available in the state and the mix of production costs. Moreover, the planner faces a general tradeoff between alleviating the markup distortion by supporting greater resilience and safety and mitigating the business-stealing externality, which exceeds the consumer-surplus externality with general HSA preferences that satisfy MSLD. This tradeoff will vary across states of the world and will reflect the exact form of preferences, the various preference parameters, and the sizes of the cross-country differences in costs and riskiness.

In the next section, we resort to numerical methods to explore some of these tradeoffs. Before that, we return briefly to the special case of symmetric CES preferences, for which a strong characterization of the second-best policies is possible even with asymmetric costs and risks. With CES preferences, the price index plays a dual role as both welfare metric and demand aggregator, as we have noted before. We show in the appendix that this exceptional feature of the CES implies that the constrained optimum is characterized by $\Pi_h = \Pi_f$, much like the *laissez-faire* equilibrium. That is, the planner has no reason to tilt supplier relationships toward one location or the other. This means that the second best can be achieved with a single policy instrument, namely a tax or subsidy for diversification. However, as we also show in the appendix, point O always lies *below* point E on the $\Pi_h = \Pi_f$ curve, so, with CES preferences, it is always desirable for the government to promote resiliency with a subsidy for all values of the cost and risk parameters. The explanation is the same as in the symmetric case; with CES preferences, the consumer-surplus externality and the business-stealing externality exactly offset one another *in every state of the world.* What remains are the distortions in every state that result from the fixed and positive markup of consumer prices over marginal costs. The constrained policy maker that cannot eliminate the consumption distortions directly with a fixed consumption subsidy at rate $(\sigma-1)/\sigma$ can instead partially alleviate the distortion by promoting greater product availability in all states of the world.

We summarize our findings about second-best supply-chain policy in

Proposition 2 *Suppose that consumption subsidies are infeasible. If consumers have symmetric CES preferences, the planner can achieve a constrained optimum with a single policy instrument, namely a subsidy for diversification ($\varphi_b > 0$). If consumers have symmetric translog preferences and the input costs and disruption risks in the two countries are symmetric, the constrained optimum can again be achieved with a single policy, which must be a tax on diversification if* $\varepsilon < \frac{\theta\rho(2+\theta\rho)}{2(2+3\theta\rho)}$

and a subsidy for diversification if $\varepsilon > \frac{\theta\rho(2-\rho)[1+\theta\rho(2-\rho)]}{1+3\theta\rho(2-\rho)}$. *In other circumstances, two policies instruments are generally needed to alter both the incentives for diversification and the incentives for sourcing at home versus abroad.*

6 Numerical Exploration of the Constrained Optimum

When monopolistically-competitive firms form their supply chains with an eye to potential disruptions, the market equilibrium features several sources of inefficiency. The consumer-surplus externality associated with product availability suggests underinvestment in resilience, whereas the business-stealing externality implies just the opposite. Meanwhile, monopoly pricing generates insufficient consumption of differentiated products relative to the numeraire good in realistic situations when fiscal policies cannot be used to align relative prices with relative marginal costs. We have been able to characterize the policy imperatives that these distortions create under CES preferences and, with more general HSA preferences, when the home and foreign suppliers are similar with respect to costs and risks. Armed with our understanding of the nature of the distortions, we turn now to numerical methods to explore the constrained optimal policies in situations when costs and risks differ in the two countries. To this end, we henceforth assume that preferences take the symmetric translog form.

Figure 4 depicts the constrained optimal fractions of firms (on the left) and the policy wedges at the second-best allocation (on the right) for two different values of ε, the elasticity of demand for differentiated products. The figure is drawn for the case when production costs are similar in the two countries ($q_H = q_F$), but we show in the appendix (see Figures 6 and 7) that qualitatively similar patterns emerge when costs differ. Figure 4 illustrates the comparative statics of the equilibrium, constrained optimum and optimal supply-chain policies with respect to variation in the cross-country risk differential.

In panels (a) and (b), we see the outcomes for a relatively low value of ε, namely $\varepsilon = 1.2$. Panel (a) shows the fraction of firms that choose each of the investment strategies in the laissez-faire equilibrium (solid curves) and in the constrained optimum (dashed curves). When $\gamma_H = \gamma_F$, at the left side of the panel, the market equilibrium features excess investment in resilience ($\mu_b > \mu_b^*$) and insufficient investment in exclusive supply relationships ($\mu_h^* = \mu_f^* = \mu^* > \mu$). These numerical outcomes mirror the theoretical results from Section 5. They reflect the fact that, under MSLD, the business-stealing effect dominates the consumer surplus effect. Moreover, with ε relatively small, the consumption distortion caused by markup pricing is not too severe. In panel (b), we see that $w_h^* = w_f^* = w^* < 0$, so the constrained optimum can be achieved either with a tax on firms that diversify, or with equal subsidies to firms that invest in exclusive supply relationships either at home or abroad. As we increase the risk of disruption in F—so that the "safety premium" in the home country rises—both the competitive equilibrium and the constrained optimum are characterized by greater fractions of diversified firms, greater fractions of firms that form relationships only onshore, and smaller fractions of firms that form relationships only abroad. These findings are intuitive, but

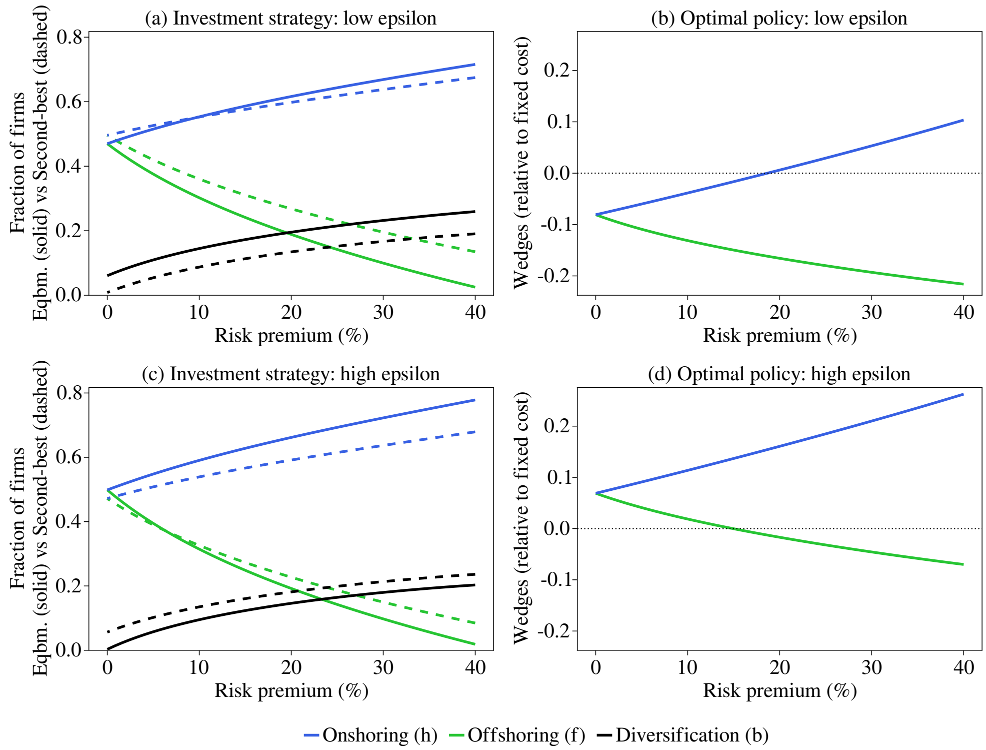

Figure 4: Second-Best Policies: Risk Differences Across Locations

Note: Baseline simulation has $\gamma_H = \gamma_F = 0.9$, $q_H = q_F = 0.1$, $\theta = 8.0$, and $\rho = 0.7$. Low and high epsilon correspond to $\varepsilon = 1.2$ and $\varepsilon = 1.7$ respectively. Fixed cost chosen so that $\min(\mu_b^*, \mu_b^o) \approx 0$ in the baseline symmetric simulation. This yields $k = 0.13$ for $\varepsilon = 1.2$ and $k = 0.37$ for $\varepsilon = 1.7$. The risk premium is computed as $-(\gamma_F - \gamma_H)/\gamma_H$, where we keep γ_H constant at its baseline value.

what is less obvious is what happens to the wedges between social and private incentives. In panel (b) we see that w_h^* rises while w_f^* falls. This implies, for example, that second-best subsidies for exclusive offshore relationships grow (φ_f^*) while subsidies for sourcing relationships at home (φ_h^*) shrink, if the planner eschews taxes or subsidies on diversification ($\varphi_b^* = 0$).

How do we understand this finding? In panel (a) in Figure 4, we see that the fraction of firms with sourcing relationships exclusively in the home country rises above the fraction with sourcing relationships exclusively offshore, as we would expect when home suppliers become relatively more reliable than foreign suppliers. The increase in product diversity and in competition in state H relative to state F generates a decline in the price index P^H relative to P^F. But the monopoly distortion is more severe when the price index is high, so the consumption shortfall is greater in state F than in state H. The planner wishes to combat the higher price index in state F with a policy that tilts sourcing toward the foreign country.

As the foreign country becomes even riskier, the planner continues to discourage diversification; we continue to find the second-best fraction of diversified firms, μ_b^*, below the free-market level. But the social cost of the market's misallocation between home sourcing and foreign sourcing also grows, and so the gap between the two wedges w_h^* and w_f^* widens. At some risk differential close to 20% in the figure, the planner's desire to shift the location of exclusive sourcing from the home country to the foreign country implies a second-best *tax* on onshore relationships, combined with an even larger subsidy for investing in a single relationship abroad.[21] The net effect of the second-best policies is to raise μ_f at the expense of both μ_h and μ_b.

The situation is similar for larger values of ε, such as depicted in panels (c) and (d) of Figure 4, except in one important respect. With a more elastic demand for differentiated products, the misallocation generated by markup pricing weighs more heavily in the planner's calculus compared to the net effect of the business-stealing and consumer-surplus externalities. The optimal policy in the symmetric environment entails a net subsidy to diversification, which can be achieved with $\varphi_b > 0 = \varphi_h = \varphi_f$ or with $\varphi_h = \varphi_f < 0 = \varphi_b$. As the risk differential grows, the planner once again tilts policy in favor of exclusive sourcing relationships abroad, to offset the increasingly deleterious effects of under-consumption of differentiated products when foreign supply is disrupted. For a sufficiently great probability of supply disruption in the foreign country, the planner subsidizes strategy f, while still ensuring that the net effect of supply chain policy is to induce more diversification and greater resilience.

Notice too the scale of the optimal subsidies. Recall from Section 4 that, when able to implement the optimal, state-and-product-contingent consumption subsidies, the planner taxes diversification (or subsidizes the two strategies involving exclusive relationships) at 100% of the fixed cost k, regardless of the configuration of cost and risk parameters.[22] In the second best described here, the impetus to tax diversification in order to dampen incentives for business stealing is offset by an urge to subsidize diversification to stimulate consumption of differentiated goods. The offsetting forces result in second-best policies that are an order of magnitude smaller than in the first best.

Figure 5 depicts the comparative statics with respect to foreign production costs, holding riskiness in the two locations constant (and, in this figure, equal to one another).[23] The symmetric equilibrium again requires a second-best tax on diversification when ε is small (top panels) and a subsidy when ε is larger (bottom panels). A fall in the cost of producing inputs abroad, which expands the cost discount in the offshore location, reduces the price index in state F relative to that in state H.[24] Thus, the social benefit from promoting consumption in state H comes to exceed that in state F. In panel (b), the planner alters the composition of exclusive supply relationships by offering a smaller subsidy for offshore sourcing than for onshore sourcing. For a high enough cost discount, the wedge for offshore relationships actually turns positive. In panel (c), with more elastic demand, the planner encourages resilience at the expense of exclusive relationships at home and abroad. As seen in panel (d), the taxes on single relationships that are used to encourage di-

[21] Alternatvely, the planner can achieve the same allocation with a subsidy for diversification and an even larger subsidy for offshore relationships (and $\varphi_h = 0$), so that μ_f grows at the expense of both μ_h and μ_b.

[22] This statement applies to situations, as here, with symmetric translog preferences

[23] The parameters used for this figure are the same as for Figure 4, so the baseline (symmetric) outcome is the same at the left-most point in both figures.

[24] In this case there are two reasons: more firms choose strategy f than strategy h and hence the market is more competitive in state F than in state H; and products containing inputs produced in F bear a lower cost than those produced in H.

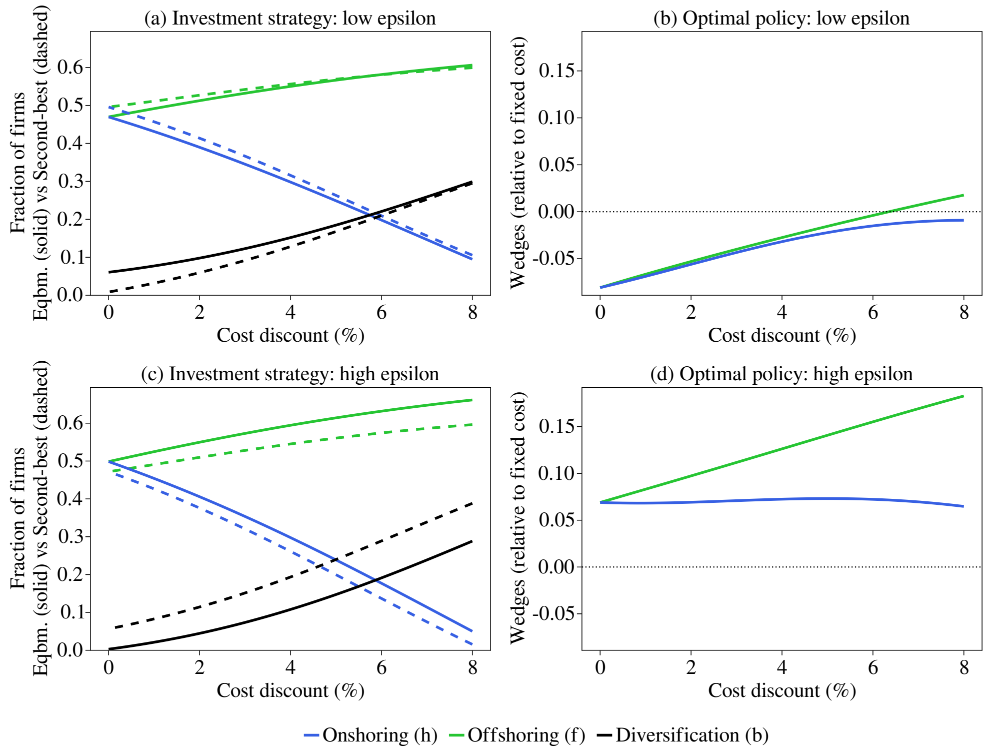

Figure 5: Second-Best Policies: Cost Differences Across Locations

Note: Baseline simulation has $\gamma_H = \gamma_F = 0.9$, $q_H = q_F = 0.1$, $\theta = 8.0$, and $\rho = 0.7$. Low and high epsilon correspond to $\varepsilon = 1.2$ and $\varepsilon = 1.7$ respectively. Fixed cost chosen so that $\min(\mu_b^\star, \mu_b^o) \approx 0$ in the baseline symmetric simulation. This yields $k = 0.13$ for $\varepsilon = 1.2$ and $k = 0.37$ for $\varepsilon = 1.7$. The cost discount is computed as $-(q_F - q_H)/q_H$, where we keep q_H constant at its baseline value.

versification diverge; onshore relationships face a lower tax than their offshore counterparts, again because the consumption distortion is more harmful in state H than in state F.

When both cost and risk parameters differ in the two possible locations for producing inputs, the ranking of the price index in states H and F is less clear-cut. Greater risk of supply disruption in the foreign country discourages private investment there, contributing to a relatively higher price index in state F. But lower production costs offshore raises the relative attractiveness of strategy f compared to strategy h, and it also has a direct effect on comparative prices in the two states reflecting the relatively lower foreign unit cost. The relative size of the wedges, w_h^* versus w_f^*, and thus the net effect on the incentives for exclusive offshoring versus exclusive onshoring, hinges on the relative strength of these forces.

7 Conclusion

Supply chain disruptions are increasingly salient and often costly. Many commentators have been quick to conclude that governments ought to be doing something to promote greater market resilience. But the welfare-theoretic calculus around government intervention is rather subtle. Private actors have a clear self-interest in taking measures to avoid disruptions to their production processes. Only when the private incentives for resilience fall short of the social benefits will government encouragement be warranted. Pointing in that direction is the observation that consumers capture part of the surplus created by the ongoing availability of firms' products. But firms also have an incentive to be in a position to reap extra profits when their rivals are suffering. The temptation for "business stealing" suggests that excess resilience is also a possible market outcome.

Surprisingly little research has addressed the desirability of government policy to promote resilience or to encourage sourcing from safer locations. In this paper, we have taken a first step. We have proposed a simple framework in which the supply of any product requires the availability of a critical input. Exogenous shocks can disrupt firms' relationships with their suppliers. We allow for idiosyncratic shocks that affect a single relationship and broader shocks that impinge on all sourcing from a particular country or region. Firms face the choice of where to develop a relationship and whether to protect their operation with backup sources of supply. We study the simplest case of two potential supply sources, one at home and one abroad and focus on a situation where domestic sourcing is costlier than sourcing abroad, but also less risky. This setting presents firms with three sourcing options: invest in a single supply relationship at home (onshoring), invest in a single relationship abroad (offshoring), or invest in supply relationships in both locations (diversification).

Since consumer gains from product availability reflect their preferences, the form of demand plays a critical role in the policy calculus. The CES demand system is popular and tractable for analysis such as ours. But it also introduces restrictions that color the findings. We allow for a CES utility function, but also for a broader class of preferences that Matsuyama and Ushchev (2017, 2020a) have developed and termed Homothetic with a Single Aggregator. The more general preferences admit non-constant markups and, in particular, application of Marshall's Second Law of Demand.

Our analysis yields several broad lessons. First, the government generally needs at least two policy instruments to achieve efficient sourcing. One instrument regulates the margin between sourcing from one location or two. The other guides the choice between sourcing at home and abroad. For example, the government might subsidize or tax supply-chain diversification, while subsidizing or taxing firms that source only at home. Or it might subsidize or tax diversification, while subsidizing or taxing offshoring. However, when preferences take the CES form, the first best can be achieved with a set of state-and-product specific consumption subsidies and with no interference in supply chain organization. The second best, in the absence of consumption subsidies, entails a subsidy to diversification. With more general forms of HSA preferences that obey Marshall's Second Law of Demand, the planner achieves the first best with consumption subsidies, a tax on firms that diversify, and a policy that tilts sourcing to one country or the other depending on the effect of relative prices on the relative strengths of the consumer-surplus externality and the business-stealing externality. In the second best, the optimal policies are qualitatively similar to the ones that achieve the first best if the elasticity of demand for differentiated products is small, but a large demand elasticity tilts the policy toward smaller taxes or even subsidies for diversification.

Our paper opens the door to further research. For example, we have considered only the simplest possible production process whereby each firm produces a final product from a single critical input. Our framework could be extended to allow for more complex supply chains, including multiple inputs from various sources that might also be combined with primary factors. We could also examine a sequential production process whereby some inputs enter into production upstream from others. Then we could ask whether the need for resilience or for safety is more important for upstream inputs or downstream inputs and how private and social incentives differ at various stages. Further, we could introduce possibilities for storage in a multi-period model, so that firms might invest in resilience by stockpiling inputs and the government might promote availability by holding reserves of consumer goods. Issues of supply chain disruption are bound to be with us for a while and many important policy questions remain to be addressed.

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Appendix

We provide in this appendix derivations of expressions discussed in the main text, as well as proofs of arguments that are not shown there. The appendix is organized according to sections in the body of the paper in order to make it easy for the reader to find these items.

Section 2

We begin by deriving expected profits $\Pi_j := \Pi_j(\boldsymbol{\mu})$, $j \in \{f, h, b\}$, where $\boldsymbol{\mu} := (\mu_h, \mu_f, \mu_b)$. To this end, recall the profit function (7),

$$\pi(\omega) = \frac{s[p(\omega)/A]}{\sigma[p(\omega)/A]} P^{1-\varepsilon}.$$

For a firm that adopts strategy j, these profits depend on the state J, $J \in \{H, F, B\}$, the costs q_H and q_F, and the vector $\boldsymbol{\mu}$. Given a state J, all firms sourcing from the same location choose the same prices. Whereas in states H and F, only sourcing from one country is feasible, in state B active firms source from two different locations.

First consider a state $J \in \{H, F\}$ in which supply chains from one country are disrupted but not so in the other country. In such a state, only firms that adopted a strategy of investing only in country J and those that invested in both countries might be able to produce, provided that their bilateral relations do not suffer an idiosyncratic shock. Each such firm pays q_J for its input. In this case, the market clearing condition (4) becomes

$$1 \equiv n^J(\boldsymbol{\mu})\, s[z^J(\boldsymbol{\mu})], \qquad J \in \{H, F\} \ , \tag{22}$$

where

$$n^J(\boldsymbol{\mu}) = (\mu_j + \mu_b)\,\rho$$

and $z^J = p^J/A^J$. These equations yield relative prices z^J in state $J \in \{H, F\}$ as functions of $\boldsymbol{\mu}$, $z^J(\boldsymbol{\mu})$.

Next note that, in state $J \in \{H, F\}$, the price index (5) can be expressed as

$$\log P^J = C_P + \log \frac{p^J}{z^J} - n^J \int_{z^J}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta,$$

where from (6),

$$p^J = \frac{\sigma(z^J)}{\sigma(z^J) - 1} q_J.$$

Using the function $z^J(\boldsymbol{\mu})$ and (22), we can express the price index P^J as a function of $z^J(\boldsymbol{\mu})$,

$$\log P^J\left[z^J(\boldsymbol{\mu})\right] := C_P + \log \frac{\sigma\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right] - 1} + \log \frac{q_J}{z^J(\boldsymbol{\mu})} - \frac{1}{s\left[z^J(\boldsymbol{\mu})\right]} \int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta, \ J \in \{H, F\}. \tag{23}$$

This function, together with (7) and $z^J(\boldsymbol{\mu})$, can be used to compute the profits of an active firm in state J, which are

$$\pi^J\left[z^J(\boldsymbol{\mu})\right] := \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]} P^J\left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon}, \quad J \in \{H, F\}. \tag{24}$$

The functions $P^J(z)$ and $\pi^J(z)$, defined in (23) and (24), are decreasing in z. To see this, differentiate $\log P^J(z)$ with respect to z, which yields[25]

$$\frac{1}{P^J(z)}\frac{dP^J(z)}{dz} = -\frac{\sigma'(z)}{\sigma(z)[\sigma(z)-1]} + \frac{s'(z)}{s(z)^2}\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta < 0,\ J \in \{H, F\}. \tag{25}$$

Next, differentiate $\log \pi^J(z)$ with respect to z, which gives

$$\frac{1}{\pi^J(z)}\frac{d\pi^J(z)}{dz} = -\frac{\sigma'(z)}{\sigma(z)}\frac{\sigma(z)-\varepsilon}{\sigma(z)-1} + \frac{s'(z)}{s(z)}\left[1 - \frac{\varepsilon-1}{s(z)}\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right],\ J \in \{H, F\}. \tag{26}$$

Equation (3) in the main text implies (see Matsuyama and Ushchev (2020)):

$$\frac{s(\zeta)}{\zeta} = \frac{s'(\zeta)}{1-\sigma(\zeta)}.$$

Therefore

$$\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta = \int_{z^J}^{\bar{z}} \frac{-s'(\zeta)}{\sigma(\zeta)-1} d\zeta < \int_{z^J}^{\bar{z}} \frac{-s'(\zeta)}{\sigma(z^J)-1} d\zeta = \frac{s(z)-s(\bar{z})}{\sigma(z)-1} = \frac{s(z)}{\sigma(z)-1}. \tag{27}$$

Using this inequality, we obtain

$$\frac{1}{\pi^J(z)}\frac{d\pi^J(z)}{dz} < \left[\frac{s'(z)}{s(z)} - \frac{\sigma'(z)}{\sigma(z)}\right]\frac{\sigma(z)-\varepsilon}{\sigma(z)-1} < 0,\ J \in \{H, F\},$$

which we summarize in the following

Lemma 1 *The functions $P^J(z)$ and $\pi^J(z)$ are declining in z for $J \in \{H, F\}$.*

In state B, in which supply chains from both countries are viable, diversified firms prefer to source from the cheaper country F (recall that $q_F < q_H$), if they can. In this case, the number of firms that source from F and pay q_F for their inputs is $n^{B,F}(\boldsymbol{\mu}) = (\mu_f + \mu_b)\rho$. The number of firms that source from country H and pay q_H for inputs is $n^{B,H}(\boldsymbol{\mu}) = \mu_h\rho + \mu_b\rho(1-\rho)$. The market clearing condition (4) implies

$$1 \equiv n^{B,H}(\boldsymbol{\mu})\, s[z^{B,H}(\boldsymbol{\mu})] + n^{B,F}(\boldsymbol{\mu}) s[z^{B,F}(\boldsymbol{\mu})], \tag{28}$$

[25]Recall that $\sigma(z) > \varepsilon$ at our equilibrium points while $\sigma'(z) \geq 0$ and $s'(z) < 0$.

which is equation (11) in the main text. The pricing equation (6) implies

$$\frac{z^{B,H}(\boldsymbol{\mu})}{z^{B,F}(\boldsymbol{\mu})} \equiv \left\{ \frac{\sigma[z^{B,H}(\boldsymbol{\mu})]}{\sigma[z^{B,H}(\boldsymbol{\mu})]-1} \right\} \Big/ \left\{ \frac{\sigma[z^{B,F}(\boldsymbol{\mu})]}{\sigma[z^{B,F}(\boldsymbol{\mu})]-1} \right\} \frac{q_H}{q_F} , \tag{29}$$

which is equation (10) in the main text. From here, we obtain solutions to the relative prices $z^{B,i}$, $i = H, F$, as functions of the vector $\boldsymbol{\mu}$, $z^{B,i}(\boldsymbol{\mu})$, $i \in \{H, F\}$.

To derive the price index (5) for state B, first note that the pricing equation (6) implies

$$\frac{1}{A^B(\boldsymbol{\mu})} = \frac{z^{B,i}(\boldsymbol{\mu}) \left\{ \sigma[z^{B,i}(\boldsymbol{\mu})] - 1 \right\}}{q_i \sigma[z^{B,i}(\boldsymbol{\mu})]}, \quad i \in \{H, F\}.$$

Using (11), we can write

$$\log A^B(\boldsymbol{\mu}) = \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) s \left[z^{B,i}(\boldsymbol{\mu}) \right] \log \left\{ \frac{q_i}{z^{B,i}(\boldsymbol{\mu})} \frac{\sigma \left[z^{B,i}(\boldsymbol{\mu}) \right]}{\sigma \left[z^{B,i}(\boldsymbol{\mu}) \right] - 1} \right\}.$$

Now, the price index (5) can be expressed as

$$\log P^B(\boldsymbol{\mu}) := \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) s \left[z^{B,i}(\boldsymbol{\mu}) \right] \log P^B \left[z^{B,i}(\boldsymbol{\mu}) \right], \tag{30}$$

where the function $\log P^J(z)$ is defined in (23). Using this result for the price index, profits of a firm that sources from country J in state B amount to

$$\pi^{B,i}(\boldsymbol{\mu}) := \frac{s \left[z^{B,i}(\boldsymbol{\mu}) \right]}{\sigma \left[z^{B,i}(\boldsymbol{\mu}) \right]} P^B(\boldsymbol{\mu})^{1-\varepsilon}, \quad i \in \{H, F\}. \tag{31}$$

Now consider expected profits from strategy j, Π_j, $j \in \{h, f, b\}$. For a firm that invests in a single supply chain, expected profits are

$$\Pi_h = \delta^H \pi^H \rho + \delta^B \pi^{B,H} \rho - k,$$

$$\Pi_f = \delta^F \pi^F \rho + \delta^B \pi^{B,F} \rho - k,$$

where δ^J is the probability that only supply chains from country J will be available, $J \in \{H, F\}$, and δ^B is the probability that supply chains from both countries will be available. These probabilities are $\delta^H = \gamma_H (1 - \gamma_F)$, $\delta^F = \gamma_F (1 - \gamma_H)$, and $\delta^B = \gamma_F \gamma_H$. Using the profit functions (24) and (31), this yields

$$\Pi_h = \Pi_h(\boldsymbol{\mu}) := \delta^H \frac{s \left[z^H(\boldsymbol{\mu}) \right]}{\sigma \left[z^H(\boldsymbol{\mu}) \right]} P^H \left[z^H(\boldsymbol{\mu}) \right]^{1-\varepsilon} \rho + \delta^B \frac{s \left[z^{B,H}(\boldsymbol{\mu}) \right]}{\sigma \left[z^{B,H}(\boldsymbol{\mu}) \right]} P^B(\boldsymbol{\mu})^{1-\varepsilon} \rho - k, \tag{32}$$

$$\Pi_f = \Pi_f(\boldsymbol{\mu}) := \delta^F \frac{s \left[z^F(\boldsymbol{\mu}) \right]}{\sigma \left[z^F(\boldsymbol{\mu}) \right]} P^F \left[z^F(\boldsymbol{\mu}) \right]^{1-\varepsilon} \rho + \delta^B \frac{s \left[z^{B,F}(\boldsymbol{\mu}) \right]}{\sigma \left[z^{B,F}(\boldsymbol{\mu}) \right]} P^B(\boldsymbol{\mu}, \boldsymbol{q})^{1-\varepsilon} \rho - k. \tag{33}$$

For a firm that invests in supply chains in both countries, expected profits are

$$\Pi_b = \sum_{J=H,F} \delta^J \pi^J \rho + \delta^B \left[\pi^{B,F} \rho + \pi^{B,H} (1-\rho)\rho\right] - 2k.$$

A firm that adopts this strategy expects profits π^F if the supply chains survive only in country F, provided it does not suffer an idiosyncratic disruption there. Similarly, it expects profits π^H if the supply chains survive only in country H, provided it does not suffer an idiosyncratic disruption there. In case supply chains in both countries are viable, the firm expects profits $\pi^{B,F}$ if its bilateral relation survives in country F and profits $\pi^{B,H}$ if its bilateral relation in F does not survive but that in H does survive. Using (24) and (31), this yields

$$\begin{aligned}\Pi_b = \Pi_b(\boldsymbol{\mu}) := &\sum_{J=H,F} \delta^J \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]} P^J \left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon} \rho \\ &+ \delta^B \left\{\frac{s\left[z^{B,F}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,F}(\boldsymbol{\mu})\right]} + \frac{s\left[z^{B,H}(\boldsymbol{\mu})\right]}{\sigma\left[z^{B,H}(\boldsymbol{\mu})\right]}(1-\rho)\right\} P^B(\boldsymbol{\mu})^{1-\varepsilon} \rho - 2k. \qquad (34)\end{aligned}$$

Using these functions and $P^J(\boldsymbol{\mu}) := P^J\left[z^J(\boldsymbol{\mu})\right],\ \ J \in \{H, F\}$, we obtain the welfare function (8) in the main text.

Section 4

We begin by deriving the social welfare function in the presence of consumption subsidies that equate consumer prices to marginal costs according to where inputs are sourced.

First, consider the pricing problem facing a producer that pays q per unit for its inputs that faces an aggregator A and that recognizes that consumers will pay only a fraction ν of the sticker price in view of the consumption subsidy at rate $1-\nu$. Then the consumer price of the final product is νp, where p is the producer price. As noted, the government sets the subsidy so that $\nu p = q$, and firms take the subsidy rate as given. They choose the sticker price as

$$p = \arg\max_{\breve{p}} P^{1-\varepsilon} s\left(\frac{\nu \breve{p}}{A}\right)(\nu \breve{p})^{-1}(\breve{p} - q).$$

The solution to this problem yields

$$p = \frac{\sigma(p/A)}{\sigma(p/A) - 1} q.$$

Therefore, the optimal subsidy rates are

$$v\left(z^J\right) = \frac{\sigma\left(z^J\right) - 1}{\sigma\left(z^J\right)},\ J \in \{H, F\},$$

$$v\left(z^{B,i}\right) = \frac{\sigma\left(z^{B,i}\right) - 1}{\sigma\left(z^{B,i}\right)},\ i \in \{H, F\}.$$

These optimal subsidies vary across states of the world if the elasticity of substitution is not constant, and they vary in state B according to the source of the inputs embodied in the final good.

We consider outcomes with $\boldsymbol{\mu} \gg 0$. Now, the market clearing conditions (9) and (11) must still be satisfied, but (10) is replaced with

$$\frac{z^{B,H}}{z^{B,F}} = \lambda := \frac{q_H}{q_F}. \tag{35}$$

It follows that the functions $z^J(\boldsymbol{\mu})$, $J \in \{H, F\}$ are the same as before, but the functions $z^{B,H}(\boldsymbol{\mu})$ and $z^{B,F}(\boldsymbol{\mu})$ are replaced by $\tilde{z}^{B,F}(\boldsymbol{\mu})$ and $\tilde{z}^{B,H}(\boldsymbol{\mu}) \equiv \lambda \tilde{z}^{B,F}(\boldsymbol{\mu})$, where the latter are obtained as solutions to (35) and (11). In what follows, we denote with a tilde any function that arise when the consumption subsidies are in place, except for those functions—like $z^H(\boldsymbol{\mu})$ and $z^F(\boldsymbol{\mu})$—that do not change as a result of the subsidies .

With the consumption subsidies in place, firms' operating profits in the various states are

$$\tilde{\pi}^J(\boldsymbol{\mu}) := \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right] - 1} \tilde{P}^J\left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon}, \; J \in \{H, F\}, \tag{36}$$

$$\tilde{\pi}^{B,i}(\boldsymbol{\mu}) := \frac{s\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right]}{\sigma\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right] - 1} \tilde{P}^B(\boldsymbol{\mu})^{1-\varepsilon}, \; i \in \{H, F\}, \tag{37}$$

where, using (5), the price indexes are

$$\log \tilde{P}^J(\boldsymbol{\mu}) := \log \tilde{P}^J\left[z^J(\boldsymbol{\mu})\right] = C_P + \log \frac{q_J}{z^J(\boldsymbol{\mu})} - n^J(\boldsymbol{\mu}) \int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta, \quad J \in \{H, F\}, \tag{38}$$

$$\log \tilde{P}^B(\boldsymbol{\mu}) = C_P + \log \tilde{A}^B(\boldsymbol{\mu}) - \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) \int_{z^{B,i}(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta, \tag{39}$$

and $\tilde{A}^B(\boldsymbol{\mu})$ is obtained from

$$1 \equiv n^{B,H}(\boldsymbol{\mu})\, s\left[\frac{q_H}{\tilde{A}^B(\boldsymbol{\mu})}\right] + n^{B,F}(\boldsymbol{\mu})\, s\left[\frac{q_F}{\tilde{A}^B(\boldsymbol{\mu})}\right].$$

Therefore,

$$\tilde{A}^B(\boldsymbol{\mu}) \equiv \frac{q_F}{\tilde{z}^{B,F}(\boldsymbol{\mu})} \equiv \frac{q_F}{\tilde{z}^{B,F}(\boldsymbol{\mu})}. \tag{40}$$

Lump-sum taxes are levied in state J to finance the consumption subsidies paid in that state. Using the subsidy rates $v(z) = [\sigma(z) - 1]/\sigma(z)$, the required taxes are

$$\tilde{T}^H(\boldsymbol{\mu}) = -(\mu_h + \mu_b)\, \tilde{\pi}^H(\boldsymbol{\mu})\rho,$$

$$\tilde{T}^F(\boldsymbol{\mu}) = -(\mu_f + \mu_b)\, \tilde{\pi}^F(\boldsymbol{\mu})\rho,$$

$$\tilde{T}^B(\boldsymbol{\mu}) = -(\mu_f + \mu_b)\,\tilde{\pi}^{B,F}(\boldsymbol{\mu})\rho - [\mu_h + \mu_b(1-\rho)]\,\tilde{\pi}^{B,H}(\boldsymbol{\mu})\rho.$$

It follows that

$$\sum_{J=H,F,B} \delta^J \tilde{T}^J(\boldsymbol{\mu}) + \sum_{j=h,f,b} \mu_j \tilde{\Pi}_j(\boldsymbol{\mu}) = -(\mu_h + \mu_f + 2\mu_b)\,k.$$

The welfare function (8) therefore becomes

$$\tilde{W}(\boldsymbol{\mu}) = \bar{Y} + \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J \tilde{P}^J(\boldsymbol{\mu})^{1-\varepsilon} - (\mu_h + \mu_f + 2\mu_b)\,k, \tag{41}$$

as shown in (15).

We next characterize the wedges that determine optimal supply chain policies. To this end, we first derive the first-order conditions for the optimal allocation $\boldsymbol{\mu}^o \gg 0$, which are characterized by $\frac{d\tilde{W}(\boldsymbol{\mu}^o)}{d\mu_j} = 0$, $j = h, f$, where, for a general function $G(\boldsymbol{\mu})$, $dG(\boldsymbol{\mu})/d\mu_j$ is the change in $G(\cdot)$ from the variation $d\mu_j = -d\mu_b > 0$. Using the price indexes (38) and (39), together with (9) and (40), we obtain

$$\begin{aligned}
\frac{d\tilde{W}(\boldsymbol{\mu}^o)}{d\mu_j} = &- \sum_{J=H,F} \delta^J \tilde{P}^J(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \frac{dn^J(\boldsymbol{\mu}^o)}{d\mu_j} \\
&- \delta^B \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\frac{d\log \tilde{A}^B(\boldsymbol{\mu}^o)}{d\mu_j} + \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}^o) \frac{s\left[\tilde{z}^{B,i}(\boldsymbol{\mu}^o)\right]}{\tilde{z}^{B,i}(\boldsymbol{\mu}^o)} \frac{d\tilde{z}^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j}\right] \\
&+ \delta^B \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \sum_{i=H,F} \left[\int_{z^{B,i}(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \frac{dn^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j} + k = 0, \quad j \in \{h, f\},
\end{aligned}$$

Note, however, that $d\log \tilde{z}^{B,F}(\boldsymbol{\mu}^o)/d\mu_j = d\log \tilde{z}^{B,H}(\boldsymbol{\mu}^o)/d\mu_j$. Then, using (11),

$$\begin{aligned}
&\frac{d\log \tilde{A}^B(\boldsymbol{\mu}^o)}{d\mu_j} + \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) \frac{s\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right]}{\tilde{z}^{B,i}(\boldsymbol{\mu})} \frac{d\tilde{z}^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j} \\
= \quad &-\frac{d\log \tilde{z}^{B,F}(\boldsymbol{\mu})}{d\mu_j} \left[1 - \sum_{i=H,F} n^{B,i}(\boldsymbol{\mu})\, s\left[\tilde{z}^{B,i}(\boldsymbol{\mu})\right]\right] = 0.
\end{aligned}$$

In other words,

$$\frac{d\log \tilde{P}^B(\boldsymbol{\mu})}{d\mu_j} = -\sum_{i=H,F} \left[\int_{\tilde{z}^{B,i}(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \frac{dn^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j}. \tag{42}$$

The first-order conditions for the first-best allocation can therefore be written as

$$\frac{d\tilde{W}(\boldsymbol{\mu}^o)}{d\mu_j} = -\sum_{J=H,F} \delta^J \tilde{P}^J(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \frac{dn^J(\boldsymbol{\mu}^o)}{d\mu_j}$$
$$+ \delta^B \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \sum_{i=H,F} \left[\int_{\tilde{z}^{B,i}(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \frac{dn^{B,i}(\boldsymbol{\mu}^o)}{d\mu_j} + k = 0, \quad j = h, f.$$

Next use $n^F(\boldsymbol{\mu}) = (1-\mu_h)\rho$, $n^H(\boldsymbol{\mu}) = (1-\mu_f)\rho$, $n^{B,F}(\boldsymbol{\mu}) = (\mu_f + \mu_b)\rho$, and $n^{B,H}(\boldsymbol{\mu}) = [\mu_h + \mu_b(1-\rho)]\rho$ to obtain $dn^F(\boldsymbol{\mu})/d\mu_f = 0$, $dn^F(\boldsymbol{\mu})/d\mu_h = -\rho$, $dn^H(\boldsymbol{\mu})/d\mu_h = 0$, $dn^H(\boldsymbol{\mu})/d\mu_f = -\rho$, $dn^{B,F}(\boldsymbol{\mu})/\mu_f = 0$, $dn^{B,F}(\boldsymbol{\mu})/d\mu_h = -\rho$, $dn^{B,H}(\boldsymbol{\mu})/d\mu_f = -(1-\rho)\rho$, $dn^{B,H}(\boldsymbol{\mu})/\mu_h = \rho^2$ for $\boldsymbol{\mu} \gg 0$. These expressions allow us to represent $d\tilde{W}(\boldsymbol{\mu}^o)/d\mu_j = 0$ for $j \in \{h, f\}$ as

$$k = \delta^H \tilde{P}^H(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\int_{z^H(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \rho + \delta^B \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\int_{\tilde{z}^{B,H}(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] (1-\rho)\rho \tag{43}$$

and

$$k = \delta^F \tilde{P}^F(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\int_{z^F(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \rho$$
$$+ \delta^B \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \left[\int_{\tilde{z}^{B,F}(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta - \rho \int_{\tilde{z}^{B,H}(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right] \rho. \tag{44}$$

By definition,

$$w_j^o := \tilde{\Pi}_j(\boldsymbol{\mu}^o) - \tilde{\Pi}_b(\boldsymbol{\mu}^o) - \frac{d\tilde{W}(\boldsymbol{\mu}^o)}{d\mu_j}, \quad j \in \{h, f\}.$$

We therefore obtain

$$w_f^o = k - \delta^H \tilde{\pi}^H(\boldsymbol{\mu}^o)\rho - \delta^B \tilde{\pi}^{B,H}(\boldsymbol{\mu}^o)(1-\rho)\rho, \tag{45}$$

and

$$w_h^o = k - \delta^F \tilde{\pi}^F(\boldsymbol{\mu}^o)\rho - \delta^B \left[\tilde{\pi}^{B,F}(\boldsymbol{\mu}^o) - \rho\tilde{\pi}^{B,H}(\boldsymbol{\mu}^o)\right]\rho. \tag{46}$$

Next we use (36), (37) and (43) to derive

$$w_f^o = \delta^H \tilde{P}^H(\boldsymbol{\mu}^o)^{1-\varepsilon} \Phi\left[\tilde{z}^H(\boldsymbol{\mu}^o)\right]\rho + \delta^B \tilde{P}^B(\boldsymbol{\mu}^o)^{1-\varepsilon} \Phi\left[\tilde{z}^{B,H}(\boldsymbol{\mu}^o)\right](1-\rho)\rho, \tag{47}$$

where

$$\Phi(z) := \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta - \frac{s(z)}{\sigma(z) - 1},$$

which is equation (18) in the main text. Moreover, using (36), (37) and (44), we obtain

$$\begin{aligned} w_h^o &= \delta^F \tilde{P}^F (\boldsymbol{\mu}^o)^{1-\varepsilon} \Phi\left[\tilde{z}^F(\boldsymbol{\mu}^o)\right]\rho + \delta^B \tilde{P}^B (\boldsymbol{\mu}^o)^{1-\varepsilon} \Phi\left[\tilde{z}^{B,H}(\boldsymbol{\mu}^o)\right]\rho(1-\rho) \\ & \quad +\delta^B \tilde{P}^B (\boldsymbol{\mu}^o)^{1-\varepsilon} \left\{\Phi\left[\tilde{z}^{B,F}(\boldsymbol{\mu}^o)\right] - \Phi\left[\tilde{z}^{B,H}(\boldsymbol{\mu}^o)\right]\right\}\rho, \end{aligned} \tag{48}$$

which is equation (19) in the main text.

First, note that (27) implies $\Phi(z) < 0$ under Marshall's Second Law of Demand. Therefore, $w_f^o < 0$. Second,

$$\Phi'(z) = -\frac{s(z)}{z} - \frac{s'(z)}{\sigma(z)-1} + \frac{s(z)}{[\sigma(z)-1]^2}\sigma'(z) = \frac{s(z)}{[\sigma(z)-1]^2}\sigma'(z) > 0.$$

Since $\tilde{z}^{B,H}(\boldsymbol{\mu}^o) = \lambda\tilde{z}^{B,F}(\boldsymbol{\mu}^o) > \tilde{z}^{B,F}(\boldsymbol{\mu}^o)$, this implies $\Phi\left[\tilde{z}^{B,F}(\boldsymbol{\mu}^o)\right] - \Phi\left[\lambda\tilde{z}^{B,F}(\boldsymbol{\mu}^o)\right] < 0$ and, therefore, $w_h^o < 0$. These findings are summarized in

Lemma 2 *Let $\sigma'(z) > 0$ for $z \in (0, \bar{z})$. Then $w_j^o < 0$ for $j \in \{h, f\}$.*

Now consider two special cases. In the limiting case of symmetric CES preferences, σ is constant and $s(z) := \alpha z^{1-\sigma}$, where $\alpha > 0$ is a constant. In this case $\Phi(z) = 0$ for all z and thus $w_h^o = w_f^o = 0$. That is, the optimal allocation is achieved with no government intervention in the formation of supply chains; i.e., $\varphi_j = 0$ for $j \in \{h, f, b\}$.

In the case of symmetric translog preferences, $s(z) := -\theta \log z$, where $\theta > 0$ is a constant and $z \in (0, 1)$. These preferences imply

$$\int_z^1 \frac{s(\zeta)}{\zeta} d\zeta = \frac{1}{2}\frac{s(z)}{\sigma(z)-1}.$$

The first-order conditions (43) and (44) become

$$2k = \delta^H \tilde{\pi}^H(\boldsymbol{\mu}^o)\rho + \delta^B \tilde{\pi}^{B,H}(\boldsymbol{\mu}^o)(1-\rho)\rho,$$

$$2k = \delta^F \tilde{\pi}^F(\boldsymbol{\mu}^o)\rho + \delta^B \left[\tilde{\pi}^{B,F}(\boldsymbol{\mu}^o) - \rho\tilde{\pi}^{B,H}(\boldsymbol{\mu}^o)\right]\rho.$$

Combining these with (45) and (46) yields

$$w_f^o = w_h^o = -k. \tag{49}$$

That is, in the translog case, the optimal allocation is achieved by a policy that subsidizes fully the cost of all investments in single-country supply chains, i.e., $\varphi_b = 0$ and $\varphi_h = \varphi_f = k$.[26] We summarize these findings in

Lemma 3 *(a) In the limiting case of symmetric CES preferences, $w_j^o = 0$ for $j \in \{h, f\}$, which implies that $\varphi_j = 0$ for $j \in \{h, f, b\}$ induces the optimal allocation. (b) In the case of symmetric*

[26] Alternatively, the planner can tax diversification with $\varphi_b = -k$, while leaving $\varphi_h = \varphi_f = 0$.

translog preferences $w_f^o = w_h^o = -k$, which implies that $\varphi_b = 0$ and $\varphi_h = \varphi_f = k$ induces the optimal allocation.

Finally, consider the difference in the absolute sizes of the wedges. Using (47) and (48), we have

$$\begin{aligned} |w_h^o| - \left|w_f^o\right| &= \delta^H \tilde{P}^H (\boldsymbol{\mu}^o)^{1-\varepsilon} \Phi \left[\tilde{z}^H (\boldsymbol{\mu}^o)\right] \rho - \delta^F \tilde{P}^F (\boldsymbol{\mu}^o)^{1-\varepsilon} \Phi \left[\tilde{z}^F (\boldsymbol{\mu}^o)\right] \rho \\ &\quad + \delta^B \tilde{P}^B (\boldsymbol{\mu}^o)^{1-\varepsilon} \left\{\Phi \left[\tilde{z}^{B,H} (\boldsymbol{\mu}^o)\right] - \Phi \left[\tilde{z}^{B,F} (\boldsymbol{\mu}^o)\right]\right\} \rho. \end{aligned} \tag{50}$$

In the limit case $q_H \searrow q_F =: q$, the last term on the right-hand side of this equation equals zero. Moreover, the first-order conditions (43) and (44) imply

$$\delta^H \tilde{P}^H (\boldsymbol{\mu}^o)^{1-\varepsilon} \int_{z^H(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta = \delta^F \tilde{P}^F (\boldsymbol{\mu}^o)^{1-\varepsilon} \int_{z^F(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta. \tag{51}$$

Therefore,

$$|w_h^o| - \left|w_f^o\right| = \delta^H \tilde{P}^H (\boldsymbol{\mu}^o)^{1-\varepsilon} \frac{s\left[z^H (\boldsymbol{\mu}^o)\right]}{\sigma\left[z^H (\boldsymbol{\mu}^o)\right] - 1} \rho - \delta^F \tilde{P}^F (\boldsymbol{\mu}^o)^{1-\varepsilon} \frac{s\left[z^F (\boldsymbol{\mu}^o)\right]}{\sigma\left[z^F (\boldsymbol{\mu}^o)\right] - 1} \rho.$$

Using (51), this difference can be expressed as

$$|w_h^o| - \left|w_f^o\right| = \rho \delta^H \tilde{P}^H (\boldsymbol{\mu}^o)^{1-\varepsilon} \frac{\left\{\frac{s\left[z^H(\boldsymbol{\mu}^o)\right]}{\sigma\left[z^H(\boldsymbol{\mu}^o)\right]-1}\right\}\left\{\frac{s\left[z^F(\boldsymbol{\mu}^o)\right]}{\sigma\left[z^F(\boldsymbol{\mu}^o)\right]-1}\right\}}{\int_{z^F(\boldsymbol{\mu}^o)}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta} \left\{\Psi\left[z^F (\boldsymbol{\mu}^o)\right] - \Psi\left[z^H (\boldsymbol{\mu}^o)\right]\right\}, \tag{52}$$

where

$$\Psi(z) := \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta \Big/ \frac{s(z)}{\sigma(z) - 1}.$$

We have established

Lemma 4 *Let $q_H \searrow q_F$. Then $|w_h^o| > |w_f^o|$ if and only if $\Psi\left[z^F(\boldsymbol{\mu}^o)\right] > \Psi\left[z^H(\boldsymbol{\mu}^o)\right]$.*

Next note from (38) that with equal costs in both countries, and $n^J(\boldsymbol{\mu}) s\left[z^J(\boldsymbol{\mu})\right] = 1$,

$$\log \tilde{P}^J(\boldsymbol{\mu}) = \log \check{P}\left[z^J(\boldsymbol{\mu})\right],$$

where

$$\log \check{P}(z) := C_P + \log \frac{q}{z} - \frac{1}{s(z)} \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta.$$

It follows that $\check{P}(z) \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta$ is a declining function of z. To see this, consider

$$\frac{d}{dz} \log\left\{\check{P}(z)^{1-\varepsilon} \left[\int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta\right]\right\} = -(\varepsilon - 1) \frac{s'(z)}{s(z)^2} \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta - \frac{s(z)}{z \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta}.$$

We use

$$\frac{s'(z)}{s(z)} = -\frac{\sigma(z) - 1}{z}$$

to obtain

$$\frac{d}{dz}\log\left\{\check{P}(z)^{1-\varepsilon}\left[\int_z^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta\right]\right\} = (\varepsilon - 1)\frac{\sigma(z) - 1}{zs(z)}\int_z^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta - \frac{s(z)}{z\int_z^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta}.$$

Finally, from (27), we have

$$\int_z^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta < \frac{s(z)}{\sigma(z) - 1},$$

which implies

$$\begin{aligned}\frac{d}{dz}\log\left\{P(z)^{1-\varepsilon}\left[\int_z^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta\right]\right\} &= (\varepsilon - 1)\frac{\sigma(z) - 1}{zs(z)}\int_z^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta - \frac{s(z)}{z\int_z^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta} \\ &< -\frac{\sigma(z) - \varepsilon}{z} < 0.\end{aligned}$$

Applied to (51), this result implies

$$z^H(\boldsymbol{\mu}^o) > z^F(\boldsymbol{\mu}^o). \tag{53}$$

Therefore $|w_h^o| > |w_f^o|$ when $\Psi(z)$ is a decreasing function and $|w_h^o| < |w_f^o|$ when $\Psi(z)$ is an increasing function. We summarize this finding in

Lemma 5 *Let $q_H \searrow q_F$ and $\sigma'(z) > 0$ for $z \in (0, \bar{z})$. If $\Psi'(z) < 0$ for all $z \in (0, \bar{z})$, then $|w_h^o| > \left|w_f^o\right|$ and if $\Psi'(z) > 0$ for all $z \in (0, \bar{z})$, then $|w_h^o| < \left|w_f^o\right|$.*

Section 5

We first derive the general expressions for the wedges w_f and w_h in the constrained optimum, when consumption subsidies are not feasible. We use (8) to calculate $dW(\boldsymbol{\mu})/d\mu_j$. Evaluated at the constrained optimum $\boldsymbol{\mu}^*$, where $dW(\boldsymbol{\mu}^*)/d\mu_j = 0$, we obtain

$$\frac{dW(\boldsymbol{\mu}^*)}{d\mu_j} = \Pi_j(\boldsymbol{\mu}^*) - \Pi_b(\boldsymbol{\mu}^*) + \sum_{i=h,f,b}\mu_i\frac{d\Pi_i(\boldsymbol{\mu}^*)}{d\mu_j} - \sum_{J=H,F,B}\delta^J P^J(\boldsymbol{\mu}^*)^{1-\varepsilon}\frac{d\log P^J(\boldsymbol{\mu}^*)}{d\mu_j} = 0,\ \ j \in \{h, f\}.$$

Rearranging terms, and using the definition of the wedges in the constrained optimum, i.e., $w_j^* = \Pi_j(\boldsymbol{\mu}^*) - \Pi_b(\boldsymbol{\mu}^*)$, yields (21) in the main text. Next, from the expressions for expected profits, (32)-(34), we have

$$\frac{d\Pi_h(\boldsymbol{\mu}^*)}{d\mu_h} = \delta^B\rho\frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_h}, \qquad \frac{d\Pi_h(\boldsymbol{\mu}^*)}{d\mu_f} = \delta^H\rho\frac{\partial\pi\left[z^H(\boldsymbol{\mu}^*)\right]}{\partial z}\frac{\partial z^H(\boldsymbol{\mu}^*)}{\partial\mu_f} + \delta^B\rho\frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_f},$$

$$\frac{d\Pi_f(\boldsymbol{\mu}^*)}{d\mu_f} = \delta^B\rho\frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_f}, \qquad \frac{d\Pi_f(\boldsymbol{\mu}^*)}{d\mu_h} = \delta^F\rho\frac{\partial\pi\left[z^F(\boldsymbol{\mu}^*)\right]}{\partial z}\frac{dz^F(\boldsymbol{\mu}^*)}{d\mu_f} + \delta^B\rho\frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_h},$$

$$\frac{d\Pi_b(\boldsymbol{\mu}^*)}{d\mu_h} = \delta^F \rho \frac{\partial \pi\left[z^F(\boldsymbol{\mu}^*), q_F\right]}{\partial z} \frac{dz^F(\boldsymbol{\mu}^*)}{d\mu_h} + \delta^B \left[\rho \frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_h} + \rho(1-\rho) \frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_h}\right],$$
$$\frac{d\Pi_b(\boldsymbol{\mu}^*)}{d\mu_f} = \delta^H \rho \frac{\partial \pi\left[z^H(\boldsymbol{\mu}^*), q_H\right]}{\partial z} \frac{dz^H(\boldsymbol{\mu}^*)}{d\mu_f} + \delta^B \left[\rho \frac{d\pi^{B,F}(\boldsymbol{\mu}^*)}{d\mu_f} + \rho(1-\rho) \frac{d\pi^{B,H}(\boldsymbol{\mu}^*)}{d\mu_f}\right].$$

Substituting these derivatives into the expression for the wedges (21), we obtain

$$\begin{aligned} w_j^* = &-\delta^K \left\{ \frac{1}{\sigma[z^K(\boldsymbol{\mu}^*)]} \frac{\partial \log \pi^K[z^K(\boldsymbol{\mu}^*)]}{\partial z} - \frac{\partial \log P^K[z^K(\boldsymbol{\mu}^*)]}{\partial z} \right\} P^K[z^K(\boldsymbol{\mu}^*)]^{1-\varepsilon} \frac{dz^K(\boldsymbol{\mu}^*)}{d\mu_j} \\ &- \delta^B \left\{ \sum_K \frac{n^{B,K}(\boldsymbol{\mu}^*) s[z^{B,K}(\boldsymbol{\mu}^*)]}{\sigma[z^{B,K}(\boldsymbol{\mu}^*)]} \frac{d \log \pi^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} - \frac{d \log P^B(\boldsymbol{\mu}^*)}{d\mu_j} \right\} P^B(\boldsymbol{\mu}^*)^{1-\varepsilon}, \end{aligned} \tag{54}$$

where $K = F$ if $j = h$ and $K = H$ if $j = f$. The first term on the right-hand side of (54) represents the net externality in state K, i.e., the business-stealing externality combined with the consumer-surplus externality. The second term represents the net externality in state B.

To compute these wedges, we need explicit expressions for the partial derivatives in (54). First note that the expressions for the semi-elasticities of the price index and profits in state $K \in \{H, F\}$ are given by (25) and (26), respectively. For state B, differentiate the expression for relative prices (29) to obtain

$$\frac{d \log z^{B,H}(\boldsymbol{\mu})}{d\mu_j} \bigg/ \frac{d \log z^{B,F}(\boldsymbol{\mu})}{d\mu_j} = \left\{1 - z^{B,F}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,F}(\boldsymbol{\mu})]}{\partial z}\right\} \bigg/ \left\{1 - z^{B,H}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,H}(\boldsymbol{\mu})]}{\partial z}\right\},$$

where $\eta(z) := \sigma(z)/(\sigma(z) - 1)$ is the markup factor. Together with the market clearing condition (11), we obtain

$$\frac{d \log z^{B,K}(\boldsymbol{\mu})}{d\mu_h} = -\frac{\rho s[z^{B,F}(\boldsymbol{\mu})] - \rho^2 s[z^{B,H}(\boldsymbol{\mu})]}{\phi(\boldsymbol{\mu}) \left\{1 - z^{B,K}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,K}(\boldsymbol{\mu})]}{\partial z}\right\}}, \;\; K \in \{H, F\} \tag{55}$$

and

$$\frac{d \log z^{B,K}(\boldsymbol{\mu})}{d\mu_f} = -\frac{\rho(1-\rho) s[z^{B,H}(\boldsymbol{\mu})]}{\phi(\boldsymbol{\mu}) \left\{1 - z^{B,K}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,K}(\boldsymbol{\mu})]}{\partial z}\right\}}, \;\; K \in \{H, F\}, \tag{56}$$

where

$$\phi(\boldsymbol{\mu}) := \sum_{L=H,F} n^{B,L}(\boldsymbol{\mu}) s\left[z^{B,L}(\boldsymbol{\mu})\right] \left\{ \frac{\sigma[z^{B,L}(\boldsymbol{\mu})] - 1}{1 - z^{B,L}(\boldsymbol{\mu}) \frac{\partial \log \eta[z^{B,L}(\boldsymbol{\mu})]}{\partial z}} \right\}.$$

Differentiating the price index in state B, (30), we obtain

$$
\begin{aligned}
\frac{d\log P^B(\boldsymbol{\mu})}{d\mu_h} = & \frac{d\log z^{B,H}(\boldsymbol{\mu})}{d\mu_h}\frac{\partial \log \eta[z^{B,H}(\boldsymbol{\mu})]}{\partial \log z} \\
& + n^{B,F}(\boldsymbol{\mu})s[z^{B,F}(\boldsymbol{\mu})]\left[\frac{d\log z^{B,F}(\boldsymbol{\mu})}{d\mu_h} - \frac{d\log z^{B,H}(\boldsymbol{\mu})}{d\mu_h}\right] \\
& + \rho\left[\int_{z^{B,F}(\boldsymbol{\mu})}^{z^{B,H}(\boldsymbol{\mu})} \frac{s(\zeta)}{\zeta}d\zeta + (1-\rho)\int_{z^{B,H}(\boldsymbol{\mu})} \frac{s(\zeta)}{\zeta}d\zeta\right],
\end{aligned} \tag{57}
$$

and

$$
\begin{aligned}
\frac{d\log P^B(\boldsymbol{\mu})}{d\mu_f} = & \frac{d\log z^{B,F}(\boldsymbol{\mu})}{d\mu_f}\frac{\partial \log \eta[z^{B,F}(\boldsymbol{\mu})]}{\partial \log z} \\
& + n^{B,H}(\boldsymbol{\mu})s[z^{B,H}(\boldsymbol{\mu})]\left[\frac{d\log z^{B,H}(\boldsymbol{\mu})}{d\mu_f} - \frac{d\log z^{B,F}(\boldsymbol{\mu})}{d\mu_f}\right] \\
& + \rho(1-\rho)\int_{z^{B,H}(\boldsymbol{\mu})} \frac{s(\zeta)}{\zeta}d\zeta.
\end{aligned} \tag{58}
$$

Finally, the change in profits is given by

$$
\begin{aligned}
\frac{d\log \pi^{B,K}(\boldsymbol{\mu})}{d\mu_j} = & -\left(\sigma[z^{B,K}(\boldsymbol{\mu})] - 1 + \frac{\partial \log \sigma[z^{B,K}(\boldsymbol{\mu})]}{\partial \log z}\right)\frac{d\log z^{B,K}(\boldsymbol{\mu})}{d\mu_j} \\
& -(\varepsilon - 1)\frac{d\log P^B(\boldsymbol{\mu})}{d\mu_j},\ j \in \{h,f\},\ K \in \{H,F\}.
\end{aligned} \tag{59}
$$

To better understand these expressions, we consider the symmetric limiting case where $q_H \searrow q_F = q$ and $\gamma_H \searrow \gamma_F = \gamma$. In this setting, $\delta^F \to \delta^H = \delta$, and $z^{B,F}(\boldsymbol{\mu}) \to z^{B,H}(\boldsymbol{\mu}) =: z^B(\boldsymbol{\mu})$. As a result, the expression for the wedge (54) becomes

$$
\begin{aligned}
w_j^* = & -\delta\left\{\frac{\frac{\partial \log \pi[z^K(\boldsymbol{\mu}^*)]}{\partial z}}{\sigma\left[z\left(\boldsymbol{\mu}^*\right)\right]} - \frac{\partial \log P[z^K(\boldsymbol{\mu}^*)]}{\partial z}\right\} P[z^K(\boldsymbol{\mu}^*)]^{1-\varepsilon}\frac{dz^K(\boldsymbol{\mu}^*)}{d\mu_j} \\
& - \delta^B\left\{\frac{\frac{\partial \log \pi[z^B(\boldsymbol{\mu}^*),q]}{\partial z}}{\sigma\left[z^B\left(\boldsymbol{\mu}^*\right)\right]} - \frac{\partial \log P[z^B(\boldsymbol{\mu}^*)]}{\partial z}\right\} P[z^B(\boldsymbol{\mu}^*)]^{1-\varepsilon}\frac{dz^B(\boldsymbol{\mu}^*)}{d\mu_j}.
\end{aligned} \tag{60}
$$

The term $\partial \log P/\partial z$ represents the consumer-surplus externality, whereas the term $\left(\partial \log \pi/\partial z\right)/\sigma$ represents the business-stealing externality.

Before considering the signs of the wedges, we need to show that $\mu_h^* = \mu_f^*$. Recall that the necessary first-order conditions for an interior allocation are

$$
W_j(\boldsymbol{\mu}^*) = \Pi_j(\boldsymbol{\mu}^*) - \Pi_b(\boldsymbol{\mu}^*) - w_j^* = 0, \quad j = h, f.
$$

For these two necessary conditions to hold jointly, it must be that

$$\Pi_h(\boldsymbol{\mu}^*) - w_h^* = \Pi_f(\boldsymbol{\mu}^*) - w_f^*.$$

Together, the expressions for the wedges and expected profits indeed imply that $\mu_h^* = \mu_f^*$.[27]

With this result in mind, we turn to the wedges. Since $\mu_h^* = \mu_f^* =: \mu^*$, the wedges for the two sole-sourcing strategies are equal, i.e., $w_h^* = w_f^* =: w^*$. Furthermore, from (60), we have

$$w^* > 0 \quad \text{if} \quad \frac{\partial \log \pi(z)}{\partial z} > \sigma(z) \frac{\partial \log P(z)}{\partial z} \quad \text{for } z \in \{z^K(\mu^*), z^B(\mu^*)\},$$

and

$$w^\star < 0 \quad \text{if} \quad \frac{\partial \log \pi(z)}{\partial z} < \sigma(z) \frac{\partial \log P(z)}{\partial z} \quad \text{for } z \in \{z^K(\mu^*), z^B(\mu^*)\},$$

which follows from the fact that z^K and z^B are decreasing in μ_j. General HSA preferences do not yield simple parametric conditions that satisfy these inequalities. But we can gain further insight by considering the special cases of CES preferences and translog preferences.

First, with symmetric CES preferences, $s(z) = \alpha z^{1-\sigma}$ and $\sigma(z) = \sigma$ is a constant. Using (25) with this market-share function, the consumer-surplus externality becomes

$$\frac{\partial \log P(z)}{\partial z} = \frac{s'(z)}{s(z)} \frac{1}{\sigma - 1} < 0.$$

Next, using (26), the business-stealing externality simplifies to

$$\frac{\partial \log \pi(z)}{\partial z} = \frac{s'(z)}{s(z)} \frac{\sigma - \varepsilon}{\sigma - 1} < 0.$$

Together they imply

$$\frac{\partial \log \pi(z)}{\partial z} - \sigma \frac{\partial \log P(z)}{\partial z} = -\frac{s'(z)}{s(z)} \frac{\varepsilon}{\sigma - 1} > 0 \text{ for all } z.$$

We have established

Lemma 6 *Let $q_H \searrow q_F$, $\gamma_H \searrow \gamma_F$, and let consumers hold symmetric CES preferences. Then, $w_h^* = w_f^* > 0$.*

Turning to symmetric translog preferences, let $s(z) = -\theta \log(z)$ for $z \in (0, 1)$. Now (25) implies

$$\frac{\partial \log P(z)}{\partial \log z} = \frac{1}{\log z - 1} - \frac{1}{2}$$

[27] The first order conditions are sufficient if W is globally concave. Yet, proving the global concavity of W for general HSA preferences turns out to be a tricky task. In the NBER working paper version of this paper, Grossman et al. (2021), we show that W is concave in the CES case. We also show the global optimality of $\mu_f^* = \mu_h^*$ under symmetric translog preferences.

while (26) implies

$$\frac{\partial \log \pi(z)}{\partial \log z} = \left(1 - \frac{1}{\log z} - \varepsilon\right)\left(\frac{1}{\log z - 1} - \frac{1}{2}\right) + \frac{1}{2\log z}.$$

Together, these two expressions imply

$$\frac{\partial \log \pi(z)}{\partial \log z} - \sigma(z)\frac{\partial \log P(z)}{\partial \log z} = \varepsilon + \frac{1}{\log z}\frac{\log z - 1}{\log z - 3}.$$

Under symmetric translog preferences, the adding up constraints of market shares generate relative prices $\log z^J(\mu) = -1/[\theta n(\mu)]$ for $n^J(\mu) = n(\mu) := \rho(1-\mu)$, $J \in \{H, F\}$, and $\log z^B(\mu) = -1/[\theta n^B(\mu)]$ for $n^B(\mu) := \rho[1 + (1-\rho)(1-2\mu)]$. It follows that

$$\frac{\partial \log \pi[z(\mu^*)]}{\partial z} > \sigma[z(\mu^*)]\frac{\partial \log P[z(\mu^*)]}{\partial z} \quad \Longleftrightarrow \quad \varepsilon > \theta n^K(\mu^*)\frac{1 + \theta n^K(\mu^*)}{1 + 3\theta n^K(\mu^*)}, \quad K \in \{H, F, B\}.$$

Finally, we note that $n^B(\mu) > n(\mu)$ for $\mu < 1/2$, and that the product $x(1+x)/(1+3x)$ is increasing in x. We conclude that

$$\varepsilon < \theta n(\mu^*)\frac{1 + \theta n(\mu^*)}{1 + 3\theta n(\mu^*)} \Longrightarrow w^* < 0,$$

and

$$\varepsilon > \theta n^B(\mu^*)\frac{1 + \theta n^B(\mu^*)}{1 + 3\theta n^B(\mu^*)} \Longrightarrow w^* > 0.$$

Although the values of $n(\mu^*)$ and $n^B(\mu^*)$ are endogenous, it is possible to derive parametric restrictions that guarantee that one or the other of these inequalities holds. Specifically, if $\varepsilon < \theta n(\mu^*)\frac{1+\theta n(\mu^*)}{1+3\theta n(\mu^*)}$ holds for the smallest possible value of n, then it must hold for all n. Therefore $w^* < 0$ if $\varepsilon < \theta\rho(2+\theta\rho)/2(2+3\theta\rho)$. Similarly, if $\varepsilon > \theta n^B(\mu^*)\frac{1+\theta n^B(\mu^*)}{1+3\theta n^B(\mu^*)}$ holds for the largest possible value of n^B, then it must hold for all n. Therefore $w^* > 0$ if $\varepsilon > \theta\rho(2-\rho)(1+\theta\rho(2-\rho))/(1+3\theta\rho(2-\rho))$.[28]

Lemma 7 *Let $q_H \searrow q_F$, $\gamma_H \searrow \gamma_F$ and suppose that consumers have symmetric translog preferences. Then*

$$\varepsilon < \frac{\theta\rho(2+\theta\rho)}{2(2+3\theta\rho)} \Longrightarrow w^* < 0,$$

and

$$\varepsilon > \frac{\theta\rho(2-\rho)([1+\theta\rho(2-\rho)]}{1+3\theta\rho(2-\rho)} \Longrightarrow w^* > 0.$$

To conclude this section, we return to the special case of CES preferences to show that Lemma 6 generalizes to settings with asymmetric costs and risks. Returning to (54), we have already shown that the first term in parenthesis is negative. In state B, constant mark-ups simplify equations (55)

[28]Technically, we also need to ensure that $\min\{\sigma[z(\mu^*)], \sigma[z^B(\mu^*)]\} = \sigma[z(\mu^*)] = 1 + \theta n(\mu^*) > \varepsilon$. This is not a concern for the sufficient condition $\varepsilon < \theta n(\mu^*)\frac{1+\theta n(\mu^*)}{1+3\theta n(\mu^*)}$. Regarding $\varepsilon > \theta n^B(\mu^*)\frac{1+\theta n^B(\mu^*)}{1+3\theta n^B(\mu^*)}$, a sufficient condition for $\sigma[z(\mu^*)] > \varepsilon$ for all μ^* is $1 + \theta n(1/2) = 1 + \theta\rho/2 > \varepsilon$ since n is decreasing in μ.

and (56) to

$$\frac{d\log z^{B,K}}{d\mu_h} = -\frac{\rho\left\{s[z^{B,F}(\boldsymbol{\mu}^*)] - \rho s[z^{B,H}(\boldsymbol{\mu}^*)]\right\}}{\sigma-1} < 0,\ K \in \{H, F\},$$

and

$$\frac{d\log z^{B,K}}{d\mu_f} = -\frac{\rho(1-\rho)s[z^{B,H}(\boldsymbol{\mu}^*)]}{\sigma-1} < 0,\ K \in \{H, F\}.$$

Furthermore, the semi-elasticity of the price index (57) and (58) becomes

$$\frac{d\log P^B(\boldsymbol{\mu}^*)}{d\mu_j} = -\frac{d\log z^{B,H}(\boldsymbol{\mu}^*)}{d\mu_j} = -\frac{d\log z^{B,F}(\boldsymbol{\mu}^*)}{d\mu_j} > 0,\ j \in \{h, f\}.$$

Similarly, the semi-elasticity of profits (59) becomes

$$\frac{d\log \pi^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} = -(\sigma-\varepsilon)\frac{d\log z^{B,F}(\boldsymbol{\mu}^*)}{d\mu_j} = -(\sigma-\varepsilon)\frac{d\log z^{B,H}(\boldsymbol{\mu}^*)}{d\mu_j} > 0,\ j \in \{h, f\}.$$

Combining these expressions, the second term in (54) becomes

$$\begin{aligned} &\sum_{K=H,f} \frac{n^{B,K}(\boldsymbol{\mu}^*)s[z^{B,K}(\boldsymbol{\mu}^*)]}{\sigma}\frac{d\log \pi^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} - \frac{d\log P^B(\boldsymbol{\mu}^*)}{d\mu_j} \\ = \ &\frac{\varepsilon}{\sigma}\frac{d\log z^{B,K}(\boldsymbol{\mu}^*)}{d\mu_j} < 0,\ j \in \{h, f\} \text{ and } K \in \{H, F\}. \end{aligned}$$

Then (54) implies

$$w_h^* = \rho\left(\frac{\varepsilon}{\sigma-1}\right)\left(\delta^F \pi[z^F(\boldsymbol{\mu}^*)] + \delta^B\left\{\pi[z^{B,F}(\boldsymbol{\mu}^*)] - \rho\pi[z^{B,H}(\boldsymbol{\mu}^*)]\right\}\right),$$

$$w_f^* = \rho\left(\frac{\varepsilon}{\sigma-1}\right)\left\{\delta^H \pi[z^H(\boldsymbol{\mu}^*)] + \delta^B(1-\rho)\pi[z^{B,H}(\boldsymbol{\mu}^*)]\right\}.$$

Together with the planner's first-order conditions, these expressions yield

Lemma 8 *Suppose consumers have symmetric CES preferences. Then*

$$w_h^* = w_f^* = \left(\frac{\varepsilon}{\sigma+\varepsilon-1}\right)k > 0.$$

Evidently, in the CES case, the two wedges are positive and equal to one another, which implies that the constrained optimum can be achieved with a subsidy for diversification, i.e., $\varphi_b > 0$, with $\varphi_h = \varphi_f = 0$.

Section 6

In this section, we extend the numerical results of Section 6 in the main text to include simulations with both asymmetric risks *and* costs. Figure 6 extends the comparative statics of panels

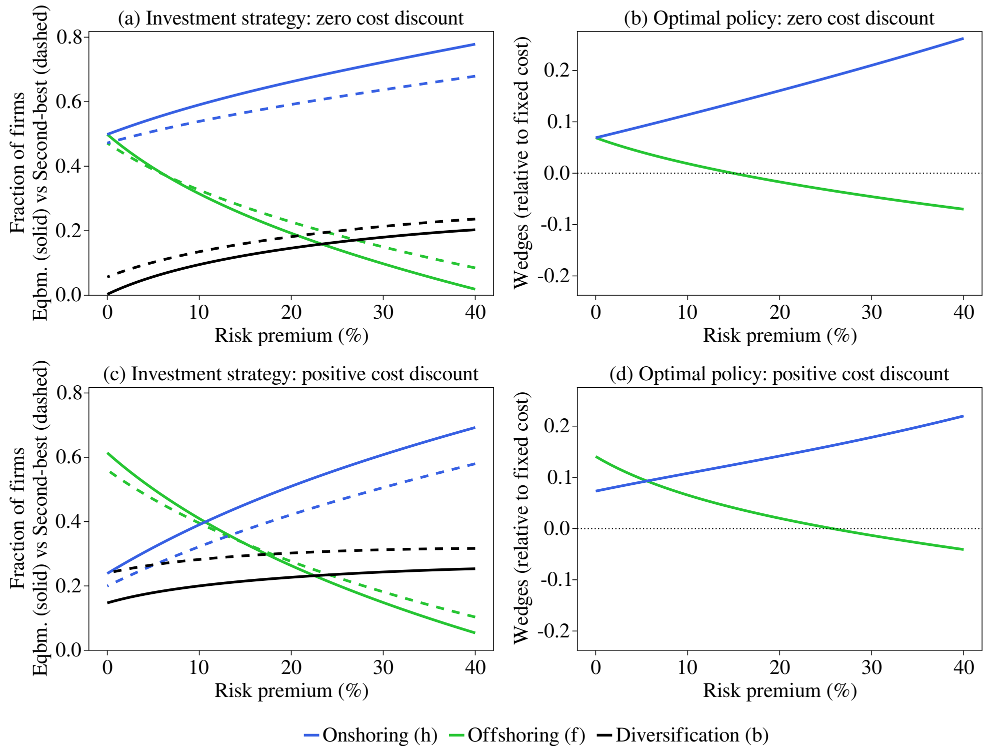

Figure 6: Second-Best Policies: Risk Differences Across Locations with Two Cost Scenarios

Note: Baseline simulation is $\varepsilon = 1.7$, $\gamma_H = \gamma_F = 0.9$, $q_H = q_F = 0.1$, $\theta = 8.0$, and $\rho = 0.7$. Fixed cost chosen so that $\min(\mu_b^\star, \mu_b^o) \approx 0$ in the baseline symmetric simulation. This yields $k = 0.37$. The risk premium is computed as $-(\gamma_F - \gamma_H)/\gamma_H$, where we keep γ_H constant at its baseline value. The cost discount in panels (b) and (d) is 5%.

(c) and (d) in Figure 4 by comparing the effect of cross-country differences in risk on the optimal supply-chain policies under two different cost discounts.[29] Panels (a) and (b) in Figure 6 depict, respectively, the fraction of firms that adopt a particular supply chain strategy and the optimal policy under the symmetric cost simulation of Figure 4. Panels (c) and (d) plot the same variables for a positive cost premium of 5%.

When the cost discount is large but the risk premium is minimal, offshoring is *ceteris paribus* more profitable than onshoring, and firms locate their supply chains disproportionately in the foreign country, both in the equilibrium and in the constrained optimum. The wedges remain positive for both strategies, although they are no longer equal. Indeed, as discussed in Section 6, when risks are identical across countries but the input cost is lower in the foreign country, the social planner wants to tax relatively more the exclusive offshore relationships as the price index is lower in state F.

As in the case with symmetric costs depicted in Figure 4, when the cost differential is positive, an increase in the risk premium is associated with a greater fraction of diversified firms, a greater fraction of firms that form relationships only onshore, and a smaller fraction of firms that form

[29]The effect of a positive cost differential on the comparative statics for the risk premium is qualitatively similar for the cases of $\varepsilon = 1.2$ and $\varepsilon = 1.7$. To conserve space, we present only the latter.

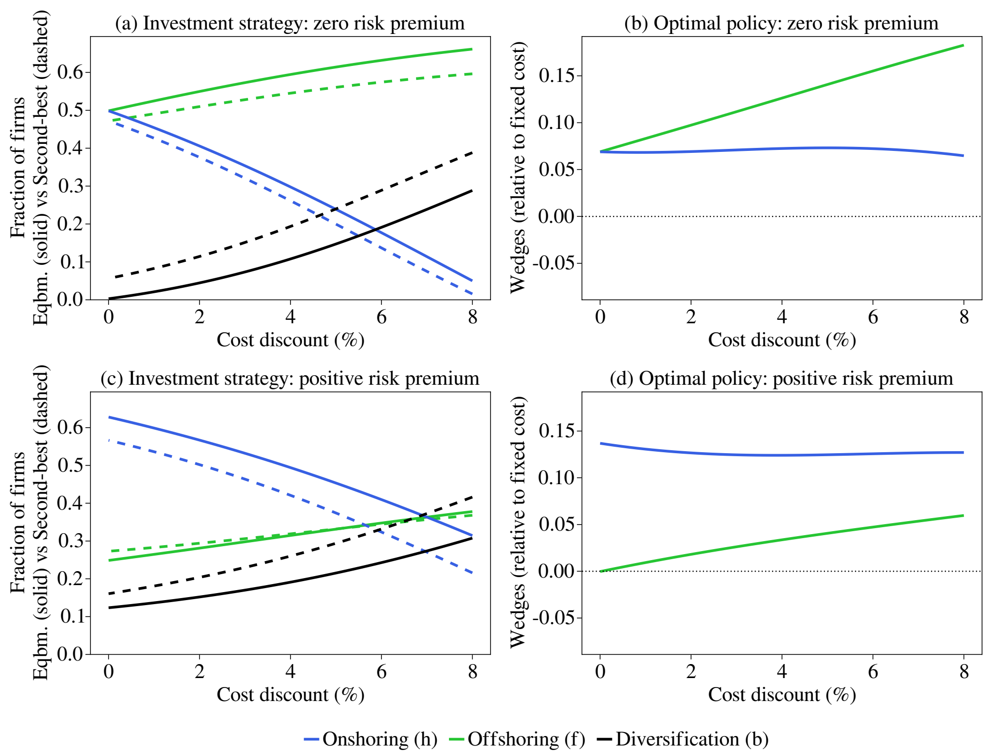

Figure 7: Second-Best Policies: Cost Differences Across Locations with Two Risk Scenarios

Note: Baseline simulation is $\varepsilon = 1.7$, $\gamma_H = \gamma_F = 0.9$, $q_H = q_F = 0.1$, $\theta = 8.0$, and $\rho = 0.7$. Fixed cost chosen so that $\min(\mu_b^\star, \mu_b^o) \approx 0$ in the baseline symmetric simulation. This yields $k = 0.37$. The cost discount is computed as $-(q_F - q_H)/q_H$, where we keep q_H constant at its baseline value. The risk premium in panels (b) and (d) is 15%.

relationships only abroad. However, in this case, firms face a tension between safe-but-expensive and riskier-but-cheaper suppliers. In panel (c), we see that for risk differentials greater than 10%, a cost discount of 5% is no longer enough to favor offshore investments, and firms locate disproportionately their supply chains in the safe-but-expensive home country.

Qualitatively, the effect of an increase in the risk premium on the optimal policies when the cost differential is positive also mimics what we have seen for symmetric costs. As the foreign risk increases, relatively more firms locate their supply chains exclusively in the home country, which triggers a relative increase in the price index in state F compared to state H, and with it an increase in w_h but a decrease in w_f. Compared with the symmetric cost simulation, the difference is now that the price index was initially lower in state F relative to state H due to the lower input cost in the foreign country. Thus, an increase in foreign risk initially shrinks the market's misallocation between home sourcing and foreign sourcing, and the wedges converge for a risk premium of 5%. Then, as the risk premium continues to grow, the price index in state F continues to increase, and the planner wants to tax relatively more the exclusive onshore relationships. Finally, for a sufficiently large risk premium, the planner's desire to shift the location of exclusive-sourcing from the home country to the foreign country implies again a tax on onshore relationships but a subsidy for investing in a single relationship abroad.

Figure 7 extends the comparative statics of panels (c) and (d) in Figure 5 by allowing for a positive risk premium. Panel (a) and (b) reproduce the results illustrated in panels (c) and (d) of the earlier figure, where $\gamma_H = \gamma_F$, while panels (c) and (d) in Figure 7 depict outcomes and policies with a positive risk premium of 15%.[30] When the cost discount is small relative to the risk premium, onshore sourcing relationships are relatively more attractive and a larger fraction of firms opt for strategy h. As the cost discount grows, the relative advantage of the foreign country increases, and a larger fraction of firms decide to form their exclusive relationship with foreign suppliers. This intuitive pattern mimics the findings for the case where risks are symmetric.

Regarding the optimal policies, the wedge for strategy f is relatively smaller than that for strategy h when the cost discount is relatively small. This echoes the discussion of Figure 4; when the risk premium is large but the cost discount is small, the monopoly distortion is more severe in state F when the price index is higher, and the planner wishes to combat the higher prices in this state with a policy that tilts sourcing towards the foreign country. As the cost discount grows further, the fraction of firms that form exclusive relationships with foreign supplier rises, which, as in the scenario with symmetric risks, reduces the social benefit from promoting consumption in state F, and thus the gap between w_f and w_h.

We have explored a large variety of parameters besides those illustrated here. In general, the optimal policies hinge on which country is more attractive for exclusive sourcing based on the tradeoff between risk and cost and the implications of these asymmetric investments on the sizes of the monopoly distortions in the various states of the world.

[30] Once again the qualitative properties of the figure are similar for $\varepsilon = 1.2$ and $\varepsilon = 1.7$, so we present only the latter.